Robust Active Damping in $LCL$-filter based Medium-Voltage Parallel Grid-Inverters for Wind Turbines

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Abstract—$LCL$-filter based grid-tie inverters require damping for current-loop stability. There are only software modifications in active damping, whereas resistors are added in passive damping. Although passive damping incurs in additional losses, it is widely used because of its simplicity. This article considers the active damping in medium-voltage parallel inverters for wind turbines. Due to cost reasons, only minimal software changes are allowed and no extra sensors can be used. The procedure must be robust against line-inductance variations in weak grids. Double-update mode is needed so the resonance frequency is under the Nyquist limit. The bandwidth reduction when using active damping is also required to be known beforehand. Moreover, the design procedure should be simple without requiring numerous trial-and-error iterations. In spite of the abundant literature, the options are limited under these circumstances. Filter-based solutions are appropriate and a new procedure for tuning the notch-filter is proposed. However, this procedure requires that the resistance of the inductors is known and a novel filter-based solution is proposed that uses lag-filters. The lag-filters displace the phase angle at the resonance frequency so that the Nyquist stability criterion is fulfilled. Simulations and experiments with a 100 kVA prototype validate the analysis.

Index Terms—$LCL$-filter, active damping, medium-voltage, voltage source converter, grid-tie inverter, line-inductance, notch-filter, lag-filter

I. INTRODUCTION

Grid-tie inverters require filters to limit the harmonic current injection to the grid. Compared to simple $L$-filters, $LCL$-filters result in lower overall inductance values. Passive damping uses resistors to achieve current-loop stability in the presence of the $LCL$-filter resonance, whereas active damping requires only modifications in the control software. In spite of the additional losses, passive damping is widely used in the industry as it requires no change in the control software [1].

Much literature is devoted to the topic of active damping with many procedures cited. The method of the virtual resistor, originally proposed in [2], modifies the control algorithm to emulate the behavior of a damping resistor without physically adding it [3], [4]. This procedure requires additional sensors which depend on the position of the virtual resistor. The utilization of observers has been proposed in [5] to avoid using extra sensors at the expense of more controller complexity and computational load. Careful design of the digital controller must be exercised because of the computational and PWM delays. The analysis of the computational delay in the feedback loop of $LCL$-filter based inverters was analyzed in [6]–[8]. State-space control allows selecting the closed-loop poles in the proper locations [9]. However, it is difficult to pick up locations that result in acceptable phase and gain margins [10]. The use of an LQR regulator [11] may alleviate the latter issue by displacing the choice to the proper weighting coefficients. Moreover, multi-variable controllers require an elevated number of computations and they are difficult to re-tune for time-varying conditions [12]. In order to overcome the line-inductance variations that de-tune the active damping procedure, adaptive control techniques were...
used in [13], [14]. Sliding mode control for \textit{LCL}-filter based inverters is proposed in [15] and in [16], which includes a Kalman filter for magnitude estimation. In [17], it was shown that the feedback of the \textit{LCL}-filter capacitor current to the voltage reference yields resonance damping. Another method for active damping consists in adding the capacitor voltage filtered by a lead-lag network [18], [19] or a high-pass filter [20] to the modulator voltage reference. Passing the reference voltage to the modulators through a notch-filter is a single loop strategy to achieve active damping [21]–[23]. The Posicast controller [24] is also an interesting technique using a single loop. The main disadvantage of these single loop strategies is that they are sensitive to changes in the resonance frequency due to line-impedance variations and may require parameter identification techniques [25] previous to being commissioned. Neural networks [26] and genetic algorithms [27] have also been applied for active damping.

The context of this article refers to \textit{LCL}-filter based medium-voltage grid-tie parallel inverters for wind turbines. These power converters usually employ passive damping because of its simplicity. The requirements for the active damping procedure in this industrial application are the following:

1) \textit{Minimum changes in the code and limited computation time}: to minimize the cost of software development and to avoid upgrading the DSP.

2) \textit{No additional sensors} apart from the converter-side sensors for current-control and protection as they are relatively costly and can incur in re-engineering expenses.

3) \textit{Robustness against line-impedance variations} as wind parks consists of parallel converters, and they are usually located in weak grids.

4) \textit{Suitable for Double-update mode}, with the sampling frequency twice the switching frequency. This widely-used technique is used to increase the current-loop bandwidth and presents aliasing problems with certain active damping procedures. In the present case, double-update mode is mandatory to make the resonance frequency be under the Nyquist limit.

5) \textit{Bandwidth assessment}, because of the limited switching frequency and so the limited bandwidth, one must know beforehand how much bandwidth will be sacrificed for using active damping.

6) \textit{Simple design}, without requiring excessive trial-and-error iterations and simple to understand without requiring intensive training hours for the company engineers.

Procedures based on the state-space [9], [10], observers [5], LQR [11], [12] and Kalman filters [16] require matrix operations and cannot meet the requirement 1). Procedures based on virtual resistors [2]–[4] require an extra sensor for each virtual resistor and cannot meet the requirement 2). Procedures based on the feedback of the capacitor voltage filtered by a lead-lag network [18], [19] and high-pass filter [20] present aliasing problems when using double update mode so that the requirement 4) cannot be met. Sliding mode controllers [15], [16] require many code changes and a DSP upgrade with faster sampling rate in order to achieve constant switching frequency so that the requirement 1) cannot be met. Adaptive control [13], [14] and neural networks [26] are sophisticated techniques that do not meet the requirement 6) simple design.

Active damping procedures based on filters at the voltage reference to the modulators are ideal candidates that meet the requirements 1), 2) and 4). The notch tuning procedure proposed in [27] uses sophisticated genetic algorithms. Reference [21] considered the notch filter, a second-order lowpass-filter, and a single lead-lag element as candidate filters. The single lead-lag element was no further considered and guidelines were provided on tuning the notch filter and the second order lag-filter. These guidelines were wide without analytical derivations so that numerous trial and error iterations were required. Reference [23] discretizes the notch filter using the bilinear transform and performs the tuning in the z-domain. The procedure loses much of the intuitiveness and becomes complex, also requiring trial and error iterations. Thus, the previous notch filter procedures do not meet the requirement 6) in various degrees. The procedure [22] involves a simple design, but the robustness against the line-inductance variations is limited and it may require estimating the grid inductance, so that the procedure does not meet the requirement 3). Finally, none of the previous procedures meets the requirement 5) with easy assessment of the available bandwidth.

In the search for an algorithm that meets all these requirements, this manuscript presents the following contributions:

1) Study on the equivalent grid inductance resulting from the parallel connection of three-phase grid-tie inverters.

2) New procedure for tuning the notch-filter that results in increased stability for line-inductance variations, meeting requirement 3). The design is simple without requiring trial-and-error iterations and it allows assessing how much bandwidth will be sacrificed for using active damping, meeting requirement 5). All the stated requirements are met, but the resistance of the inductors is required to be known.

3) Novel procedure for active damping based on lag-filters that displace the phase angle at the resonance frequency so that the Nyquist stability criterion is met. The procedure also allows assessing how much bandwidth will be sacrificed for using active damping, requirement 5). Thus, this proposed procedure meets all the stated requirements.

Fig. 1 shows a conceptual representation of the proposals stated in the article. This paper is organized as follows: Section II describes the control of the \textit{LCL}-based three-phase grid-tie parallel inverters. Section III explains the proposed new tuning procedure for the notch-filter. The novel active damping procedure based on lag-filters is explained in Section IV. Simulation results are shown in Section V and experimental results with a 100 kW prototype are shown in Section VI. Section VII discusses the results and finally, Section VIII concludes the paper.
II. CONTROL OF LCL-BASED THREE-PHASE GRID-TIE PARALLEL INVERTERS

This section explains the stability implications of connecting multiple grid-tie inverters in parallel. The section also explains the current control in the presence of additional transfer-functions in the control-loop.

Fig. 2 shows a three phase inverter connected to the grid through an LCL-filter where \( v_{dc} \) is DC-link voltage, \( C_{dc} \) the total DC-link capacitance, \( v \) the converter-side voltage, \( L \) the converter-side inductor inductance, \( R \) the converter-side inductor resistance, \( C_f \) the LCL-filter capacitor capacitance, \( i_{cf} \) the LCL-filter capacitor current, \( v_{cf} \) LCL-filter capacitor voltage, \( R_g \) the grid-side inductor resistance, \( i_g \) is the grid-side inductor current, \( L_{indg} \) the grid-side inductor inductance, \( L_{line} \) the step-up transformer leakage inductance (referred to the low-voltage side), \( v_{su} \) the voltage at the step-up transformer terminals, \( L_{line}' \) the line-inductance (referred to the high-voltage side), and \( e_g \) Grid voltage (referred to the high-voltage side). When referred to low-voltage side, the line-inductance and the grid voltage are \( L_{line} \) and \( e_g \) respectively. The grid-side inductance \( L_g = L_{indg} + L_{leak} + L_{line} \) is defined for convenience and it combines the grid-side inductor inductance, the step-up transformer leakage inductance and the line-inductance (referred to the low-voltage side). The inverter used for the simulation and experiments of this manuscript is a three-level Neutral Point Clamped (NPC) converter, but a two-level full-bridge converter can also be used.

A. Issues of Parallel Connection of Three-phase Grid-tie Inverters

In this subsection, \( n \) parallel converters connected to the grid are considered. In the following equations the resistances of all the passive elements and the presence of step-up transformer are omitted for simplicity. The subscript \( k \) refers to each of the parallel converters. All the parallel converters share a common voltage base \( V_g \) (the voltage at the PCC) and \( I^k_{g} \) is the current base for the parallel converter \( k \). If it is assumed that all the \( n \) parallel inverters have the same parameters in per unit values. The equations for each parallel converter \( k \) in per unit (base: \( V_g \) and \( I^k_{g} \)) are as follows:

\[
\hat{i}^k = \hat{v}_{cf}^k - L_{line} \hat{s} \hat{i}^k
\]  

(1a)

The variables in per unit values are underlined: \( \hat{v}^k \) is the converter-side voltage of the parallel converter \( k \) in per unit, \( \hat{v}_{cf}^k \) the LCL-filter capacitor voltage of the parallel converter \( k \) in per unit, \( \hat{v}_{cf}^k \) LCL-filter capacitor voltage of the parallel converter \( k \) in per unit, and \( \hat{v}_{line}^k \) is the line-side inductor current of the parallel converter \( k \) in per unit. The parameters in per unit are in lower case: \( l \) is the converter-side inductor inductance in per unit, \( c_f \) the LCL-filter capacitor capacitance in per unit, \( l_{indg} \) the grid-side inductor inductance in per unit, \( v_{line}^k \) the line-inductance expressed in per unit values using the bases \( V_g \) and \( I^k_{g} \), and \( e_g \) the grid voltage in per unit. Finally \( s \) is the Laplace variable. It is also assumed that all the \( n \) parallel inverters have the same controller \( c(s) \) and the same current reference per unit \( \hat{i}^k \) so that:

\[
\hat{v}^k = c(s)(\hat{s}^k - \hat{i}^k)
\]  

(2)

By analyzing (1), it can be seen that the parallel inverters behave as if they were connected to an equivalent line-inductance \( l_{lineeq}^k \) equal to the line-inductance weighted by the rated currents \( I^k_{g} \). The assumptions for the previous derivations do not consider that the switching frequency changes for different rated powers in converters so that the controllers and the parameters per unit will not be exactly the same. However, (1) gives insight into the relative influence of other parallel inverters and their rated power in the equivalent line-inductance \( l_{lineeq}^k \) as seen by the individual parallel inverter. If the base currents are exactly the same for all the parallel converters \( I^k_{g} = \text{const} \), then the overall line-inductance is \( n \) times the grid-inductance \( I_{lineeq}^k = nl_{line} \) as demonstrated by Agorreta et al. in [28].

B. Padé Approximant for Filters

Neglecting all the equivalent series resistances of the LCL-filter inductors and capacitor, the transfer function relating the inverter current \( i \) and voltage \( v \) is:

\[
G_{LCL}(s) = \frac{i}{v} = \frac{1}{Ls} \left( \frac{s^2 + \omega_{2LC}^2}{s^2 + \omega_{res}^2} \right)
\]  

(3)

with \( \omega_{2LC}^2 = (L_g C_f)^{-1} \) is the double zero of the transfer function and \( \omega_{res}^2 = L_r \omega_{2LC}^2 / L \) the double pole. The resonance frequency is \( f_{res} = \omega_{res} / (2\pi) \) [29].

In the double update mode [30], the sampling frequency \( f_s \) is twice the switching frequency \( f_s = 2f_{sw} \) and it may result in aliasing when using active damping based on magnitude feedback. Taking into account that the effects of the integral component vanishes at the resonant frequency, the phase
angles of the resonance frequency are separated by 180 degrees and their values are [31]:

\[ \varphi^+ = \frac{\pi}{2} - 3\pi \frac{f_{res}}{f_s} \quad (4a) \]
\[ \varphi^- = -3\pi \frac{f_{res}}{2} - 3\pi \frac{f_{res}}{f_s} \quad (4b) \]

where \( f_{res} \) is the resonance frequency. The angles \( \varphi^+ \) and \( \varphi^- \) are shown in the Bode plot of Fig. 8.

Fig. 3 shows the overall control of the LCL-filter based three-phase inverter. The voltage-vector oriented controller uses a PLL to detect the grid frequency \( \omega_g \). The direct current \( i_d \) is aligned with the voltage vector and it is varied to control the active power [32]. The quadrature current \( i_q \) is perpendicular to the voltage vector and it is varied to control the reactive power. The inner controllers regulate the direct \( d \) and the quadrature \( q \) currents. The outer controller that regulates the DC-link voltage \( v_d \) has the \( i_d^* \) as output. The outer controller that regulates the reactive \( q \) has the \( i_q^* \) as output. This latter controller is optional and alternatively, \( i_q^* \) can be set to zero for unity power factor.

The proposed damping procedures consists of a transfer function located after the PI controller output and before the input to the PWM modulator. This transfer function affects the low-frequency behaviour of the plant and needs to be considered in the tuning procedure of the PI controllers (this is explained in the following Subsection II-C). The bandwidth of the current-control must be lower than the resonance frequency in order to avoid interference [29], [33].

The Padé approximant \( p_{N,M}(s) \) consists of a quotient of two polynomials with degree \( N \) and \( M \) in the numerator and denominator, respectively [34]. The Padé approximant \( p_{N,M}(s) \) has the same Taylor series expansion as the function \( f(s) \) up to degree \( N + P \). The Padé approximant has been employed to approximate the low frequency behavior of transfer functions in [18], [35]. As the optimum criteria for tuning the PI controllers assumes the presence of first order systems with a single time constant, the Padé approximant \( p_{0,1}(s) \) will be considered:

\[ p_{0,1}(s) = \frac{1}{T_{Padé}s + 1} \quad p_{0,1}(0) = f(0) \quad \text{and} \quad p'_{0,1}(0) = f'(0) \quad (5) \]

The resulting Padé approximant \( p_{0,1}(s) \) has the same behaviour for the low frequency, around \( s = \omega j = 0 \), as the function \( f(s) \).

C. Tuning Procedure for the Current Controller

The PI controller is tuned by using the low-frequency model of an LCL-filter, which results from neglecting the capacitor branch [18], [29]. The resulting low-frequency model is a simple \( L \)-filter with total inductance \( L_t = L + L_g \) and the total resistance \( R_t = R + R_g \). A common procedure for tuning the current-loop is the technical optimum criterion [32], [36], [37]. This method deals with a transfer function comprising two first order systems:

\[ G_{if}(s) = \frac{K}{(\tau s + 1)(T_s s + 1)} \quad (6) \]

with the time constants so that \( T >> \tau \) and \( K \) being a gain. In (6), the large time constant is \( T = L_t/R_t \) and the smaller time constant combines the computational delay (a complete sampling cycle), the PWM delay (approximated as a half of a sampling cycle) and the time constant resulting from the Padé approximant of the transfer functions that provide active damping (notch-filter or lag-filters as explained later).

\[ \tau = T_{comp} + T_{PWM} + \tau_{Padé} = T_s + 0.5T_s + \tau_{Padé} \quad (7) \]

According to the technical optimum criterion, the integration time cancels the large time constant \( T_s = T \) and the proportional gain is set so to have 4% overshoot (so that the closed-loop transfer-function is a second-order system with \( \xi = 0.707 \) [32], a Butterworth filter with maximally flat magnitude as it corresponds to the technical optimum criterion [37]):

\[ K_p = \frac{L_t}{2\tau} \quad (8) \]

The resulting control bandwidth when using the technical optimum criterion for the PI controllers is [32]:

\[ f_{bw} = \frac{1}{2\pi} \frac{K_p}{L_t} = \left( \frac{1}{2\pi} \right) \frac{1}{2\tau} \quad (9) \]

The control bandwidth when no active damping is included \((\tau_{Padé} = 0 \text{ in } (7))\) results as follows:

\[ f_{bw}^\text{max} = \left( \frac{1}{2\pi} \right) \frac{1}{3T_s} \quad (10) \]

The presence of the transfer functions for active damping results in a bandwidth reduction because of the additional small time constant \( \tau_{Padé} \) in \( \tau \) (7). The control bandwidth when the active damping procedure is included results as follows:

\[ f_{bw}^\tau = \left( \frac{1}{2\pi} \right) \frac{1}{3T_s + 2\tau_{Padé}} \quad (11) \]
The bandwidth reduction can be calculated by dividing (10) and (11) and it results as follows:

\[
\frac{f_{bw}^{\text{max}}}{f_{bw}} = 1 + \frac{\tau_{P\text{ad}e}}{1.5T_s} \quad (12)
\]

However, the total inductance of the LCL-filter is smaller than the inductance required for an L-filter and so lower is the voltage drop. This allows lower saturation levels in the controllers during transients due to the limited DC-link voltage ceiling.

III. NEW TUNING PROCEDURE FOR THE NOTCH-FILTER

This section explains a proposed new tuning procedure for the notch filter, which results in increased stability for line-inductance variations to meet requirement 3).

The notch-filter has a narrow notch at the resonance frequency that prevents the excitation of the LCL-filter resonance. In [22], it was proposed to use \( n \) notch filters in series in order to increase the width of the notch for increased robustness:

\[
N_n(s) = \left( \frac{s^2 + 2D_s \omega_{nf}s + \omega_{nf}^2}{s^2 + 2D_p \omega_{nf}s + \omega_{nf}^2} \right)^{n_f} \quad (13)
\]

where \( \omega_{nf} \) is the notch frequency, \( D_s \) and \( D_p \) are the damping factors for the complex conjugates poles and zeros respectively and \( n_f \) is the number of sections. The Padé approximant \( p_{0,1}(s) \) of the notch-filter is:

\[
p_{0,1}(s) = \left( \frac{1}{2D_s - D_z} \right)^{n_f} \approx \frac{1}{2n_f D_p - D_z} \frac{1}{\omega_{res} s + 1} \quad (14)
\]

Therefore, the resulting delay to be considered in the tuning of the PI controllers is:

\[
\tau_{P\text{ad}e(notch)} = \frac{2n_f}{\omega_{res}} (D_p - D_z) \quad (15)
\]

The value \( n_f = 2 \) was deemed reasonable in terms of proper trade-off between notch width and computational load [22]. A tolerable bandwidth reduction can be between a half or a third, which results from selecting \( \tau_{P\text{ad}e(notch)} \) between \( 1.5T_s \) and \( 3T_s \).

The notch frequency corresponds to the resonance frequency \( \omega_n = \omega_{res} \). In [22], the damping factor \( D_z \) was selected to be zero as absolutely no component at the resonance frequency is allowed to pass through the PWM modulators. However, this criterion can be relaxed as there is always some parasitic resistance in the LCL-filter. For \( f_s/f_{res} > 2 \) by fulfilling the Nyquist frequency limit, the phase around the resonance frequency crosses \( -\pi \) as \( \varphi^+ > -\pi > \varphi^- \) according to (4). Hence, the resonance frequency is very close to the gain margin crossover frequency and the notch-filter should reduce the amplitude peak so the gain margin is positive:

\[
|C_{PI}(j\omega_{res})||G_{pd}(j\omega_{res})||N_s(j\omega_{res})| < 0.1 \quad (GM \approx 20 \text{ dB}) \quad (16)
\]

where \( G_{pd}(s) \) is the transfer function relating the inverter voltage \( v \) and current \( i \) including all the known values (or at least a lower bound) of parasitic resistive elements (inductor resistance and capacitor filter ESR) and \( C_{PI}(s) \) is the transfer function of the PI controller. A proper gain margin (GM) should be higher than 6 dB [38] and selecting 20 dB resulted in good robustness to line-inductance variations. The condition for stability is as follows:

\[
|C_{PI}(j\omega_{res})||G_{pd}(j\omega_{res})| < 0.1 \quad (GM \approx 20 \text{ dB}) \quad (17)
\]

This relaxation in selecting \( D_z \) results in a wider notch, which will increase the stability against the line-inductance variation. The Tustin rule with pre-warping at the resonance frequency reduces the notch width of the discretized filter. The matched \( z \)-transform method [39] results in a notch width closer to that of the continuous case, provided the extra reduction in gain and extra phase delay still fulfills the stability conditions.

This notch filter solution requires the resistances of the inductors to be known. There is usually no control over these parameters when designing the LCL-filter. If there is an upgrade in the inductors, the inductance will be the same but the resistance may be different (and probably lower). In addition, the resistance of the inductors varies with the temperature. Hence, a lower bound of the resistance of the inductors is necessary to apply this active damping procedure.

IV. NOVEL ACTIVE DAMPING PROCEDURE USING LAG-FILTERS

This section explains a novel procedure for active damping based on lag-filters that fulfills the conditions stated for the target application without requiring to know the resistance of the inductors.

The proposed method uses lag-filters \( G_{nlag}(s) \) to include an additional phase lag is available \( \varphi \) at the resonance frequency, so that the new phase angles at the resonance frequency are:

\[
\varphi^+ = \varphi^+ + \varphi \quad (18a)
\]

\[
\varphi^- = \varphi^- + \varphi \quad (18b)
\]

These angles are illustrated in Fig. 8. The values of \( \varphi \) required to achieve stability can be inferred from the encirclements of \(-1\) using the Nyquist criterion. However, the stability condition is easier to infer by considering that there should be a positive real part at the resonance frequency (resistive behavior \( \Re\{G_{nlag}(j\omega_{res})G_{ud}(j\omega_{res})\} > 0 \)). The phase angle at the resonance frequency is as follows:

\[
\varphi_{res} = \frac{\varphi^+ + \varphi^-}{2} + \varphi = -\pi - 3\pi \frac{f_{res}}{f_s} + \varphi \quad (19)
\]

and the necessary condition for stability is \( -\pi/2 < \varphi_{res} < \pi/2 \). Therefore, the phase delay required for stability is:

\[
-\frac{3\pi}{2} + 3\pi \frac{f_{res}}{f_s} < \varphi < -\frac{5\pi}{2} + 3\pi \frac{f_{res}}{f_s} \quad (20)
\]
Multiples of $2\pi$ have been added to (20) in order to derive the stability condition. If $\varphi_{res} = 0$ as in [40], the value of $G_{nlag}(j\omega_{res})/G_{ud}(j\omega_{res})$ is real so the behavior at the resonance frequency is purely resistive.

In order to produce the required phase delay at the resonance frequency, the proposed lag-filter uses lead-lag (zero-pole) sections. The net result is a lag compensator [38] but the zero alleviates the phase-lag at lower frequencies and so the bandwidth reduction. The number of first-order lag-filters is $n_l$ and each section will produce a phase delay $\varphi_i = \varphi/n_l$:

$$G_{nlag}(s) = \left( \frac{s + 1}{\omega_{res} + 1} \right)^{n_l}$$  \hspace{1cm} (21)

with

$$r = \sqrt{\frac{1 - \sin(\varphi_i)}{1 + \sin(\varphi_i)}}$$  \hspace{1cm} (22)

The resulting sections are discretized by using the Tustin method with pre-warping at the resonance frequency in order to preserve the desired characteristics in the discrete domain at the resonance frequency. Each lag section has unit amplitude at DC and a gain reduction of $r^2$ in the high-frequency region. The Padé approximant $p_{n,0.1}(s)$ of the lag-filters is:

$$p_{n,0.1}(s) = \left[ \frac{1}{\omega_{res} \left( r - \frac{1}{r} \right) s + 1} \right]^{n_l} \approx \frac{1}{\omega_{res}} \left( r - \frac{1}{r} \right) s + 1$$  \hspace{1cm} (23)

Therefore, the resulting delay to be considered in the tuning of the PI controllers is:

$$\tau_{Padé(nlag)} = \frac{n_l}{\omega_{res}} \left( r - \frac{1}{r} \right)$$  \hspace{1cm} (24)

The number of lag-filters $n_l$ must result in $|\varphi_i| < \pi/2$. For a fixed value of $\varphi$, a higher number of sections $n_l$ results in a smaller delay in (24) because the term depending on $r$ (22) decays faster than $n_l$. Therefore, increasing the number of sections $n_l$ will increase bandwidth by reducing (24). However, these improvements are at the expense of more computations. Making $n_l = 4$ results in approximately the same number of computations as in the previous second order notch filter and it provides a reasonable trade-off between bandwidth reduction and processing time.

The phase margin $PM$ in degrees is related to the phase delay $\varphi$ as follows (see Figs. 5 and 8):

$$\varphi = (PM - 180) \frac{2\pi}{360} + \left( -\frac{3\pi}{2} + 3\pi \frac{f_{res}}{f_s} \right)$$  \hspace{1cm} (25)

This formula can be used to select the PM for the nominal resonance frequency. However, the increase in the line-inductance $L_s$ and the increase of the number of parallel inverters increases the grid inductance $L_g$ and reduces the resonance frequency $f_{res}$. The proper tuning procedure should consider a minimum adequate $PM$ (between 30 and 60 degrees [38]) for the maximum realistic increase in the line-inductance which results in the minimum resonant frequency $f_{res}^{min}$. The proper value for $\varphi$ is calculated by substituting the selected $PM$ for the minimum resonance frequency in (25), yet the system must have a reasonable $PM$ for the nominal resonance frequency. If the resulting $PM$ for the nominal resonance frequency is too small, the worst condition $f_{res}^{min}$ cannot be fulfilled and the $PM$ should be increased.

V. Simulation Results

Table I shows all the data of the set-up used for simulations and experiments. The design procedure for the LCL-filter design is explained in [29]. The ratio $L_g/L_s = 1.6$ was selected in [29], but in this manuscript the value of the grid-side inductance $L_g$ is varied from an initial value of 0.25 mH ($L_g/L_s = 0.5$) to a final value of 2.5 mH ($L_g/L_s = 5$) in order to study the variations in the line-inductance. The simulation software is Matlab/Simulink/SimPowerSystems and the semiconductor switches are ideal. The values of the LCL-filter parameters presented in Table I result from applying the design procedure explained in [29]. The experiments will be performed in a scaled 100 kVA three-level NPC converter whereas the target industrial application uses an 800 kVA two-level converter. The voltage is measured at the filter capacitor terminals and is used for grid synchronization. For the initial value of $L_g$, the ratio between $f_s/f_{res} = 2.4$ is very close to the Nyquist limit. In addition, sudden increments in the grid inductance will be considered that may be due to additional line-inductance or due to the connection of additional parallel inverters [28]. In both situations, achieving stability is a challenging task for the active damping procedure.

A. New Tuning Procedure for the Notch-filter

Previous references [18], [22] considered a bandwidth reduction of no more than 2 when using active damping. A bandwidth reduction of 2.64 times was selected for the notch-filter procedure; simulations and experiments will show that the resulting bandwidth was sufficient for tracking the sinusoidal reference in the current-loop. Solving (12) results in $\tau_{Padé(notch)} = 2.46T_s$. The required parameters of the notch-filter $D_3 = 1.7$ and $D_2 = 8.86e - 2$ are calculated by using (15) and (17) for $n_f = 2$ as explained in Section III.

<table>
<thead>
<tr>
<th>Grid-tie inverter</th>
<th>Rated power $S_n$</th>
<th>100 kVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated ac voltage $V_n$</td>
<td>400 V</td>
<td></td>
</tr>
<tr>
<td>Rated frequency $f_n$</td>
<td>50 Hz</td>
<td></td>
</tr>
<tr>
<td>dc link voltage $V_{DC}$</td>
<td>700 V</td>
<td></td>
</tr>
<tr>
<td>Sampling frequency $f_s$</td>
<td>5100 Hz</td>
<td></td>
</tr>
<tr>
<td>PWM frequency $f_{sw}$</td>
<td>2550 Hz</td>
<td></td>
</tr>
<tr>
<td>LCL-filter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inverter coil inductance $L$</td>
<td>0.5 mH (5.67 %)</td>
<td></td>
</tr>
<tr>
<td>Inverter coil resistance $R$</td>
<td>4.7 mΩ (0.17 %)</td>
<td></td>
</tr>
<tr>
<td>Filter capacitor $C_f$</td>
<td>33 µF (2.9 %)</td>
<td></td>
</tr>
<tr>
<td>Grid coil inductance $L_g$</td>
<td>0.25 mH (2.8 %)</td>
<td></td>
</tr>
<tr>
<td>Grid coil resistance $R_g$</td>
<td>2.36 mΩ (0.09 %)</td>
<td></td>
</tr>
<tr>
<td>Resonance frequency $f_{res}$</td>
<td>2135 Hz</td>
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</tr>
</tbody>
</table>
Fig. 4: Frequency response of the continuous notch-filter, the discrete notch-filter using the Tustin rule with pre-warping at the resonance frequency and the discrete notch-filter using the matched z-transform rule.

Fig. 5 shows the Bode plot for the open-loop transfer function with no notch-filter (no active damping procedure), the notch-filter using the Tustin rule and the notch-filter using the matched z-transform rule. It can be seen that the system is unstable when no notch-filter (no active damping procedure) is included. The overall system is stable when the notch-filter using the Tustin rule is included. The gain margin crossover frequency is still approximately at the resonance frequency (as explained in Section III) and the gain margin is positive rendering the overall system stable. The overall system is also stable when the notch-filter uses the matched z-transform and it has a wider notch. This will result in increased robustness against resonance frequency variations due to changing line-impedance. Fig. 5 also shows the Bode plot for the low-frequency model that neglects the LCL-filter branch capacitor and uses the Padé approximant of the notch-filter, which is very close to the exact model in the low-frequency region.

The minimum GM and PM are also shown in Fig. 5. It can be seen that the control-loop is unstable when the active damping procedure is not present with negative GM at the resonance frequency. After including the notch filter, the GM at the resonance frequency is positive and 23.7 dB; close to the selected 20 dB in (17). The minimum GM and PM are respectively 11.4 dB and 52.4 degrees for the case using the Tustin rule. For the case of the z-transform matched rule, the minimum GM and PM are respectively 5.8 dB and 28.7 degrees. The minimum stability margins are lower for the z-transform matched rule, but still around the recommended 6 dB and 30 degrees [38].

B. Novel Active Damping Procedure based on Lag-filters

For the multiple lag-filters, a phase margin $PM = 30$ degrees is considered adequate when the grid inductance $L_g$ is nine times the initial value (this situation corresponds to the minimum resonance frequency $f_{res}^{min} = 1362.9$ Hz). This results in $\varphi = -155.7$ degrees by substituting in (25) and $\varphi_i = -38.9$ degrees ($r = 2.09$ according to (22)) by selecting $n_i = 4$ in order to have the same number of computations as in the notch-filter case.

The resulting delay $\tau_{Padé(nlag)} = 2.46 T_s$ is calculated with (24) and entails an acceptable bandwidth reduction of 2.6 times (same as in the notch-filter case). For this case as well it is required to use the low-frequency model that neglects the LCL-filter capacitor branch and uses the Padé approximant of the lag-filters in order to tune the PI controllers according to (8). Fig. 6 compares the step responses of the exact model and the low-frequency model that neglects the LCL-filter capacitor branch and uses the Padé approximant of the lag-filters (without loss of generality, the PI controller gains were tuned to result in no overshoot). The lag filters also produce some initial warping, but the step-responses are very similar with the same settling time and the approximation method is validated.
Fig. 7 shows the frequency response of the continuous lag-filters and the discrete lag-filters using the Tustin rule with pre-warping at the resonance frequency.

The current-loop is stable for all the situations, provided by these limited resistance values and active damping are used. The current-loop cannot be stable by the passive damping method. The overall system is stable and the selection of PM = 30 degrees for the maximum grid inductance was adequate. Fig. 8 also shows the Bode plot for the low-frequency model that neglects the LCL-filter branch capacitor and uses the Padé approximant of the notch-filter, which is very close to the exact model in the low-frequency region. The minimum GM and PM are respectively 6.69 dB and 32 degrees, slightly higher than the recommended 6 dB and 30 degrees [38].

Fig. 9 shows the root locus in the z-plane when varying $L_g$ from 75% to 100% of the initial value (from 0.2 mH to 2.5 mH) for the case of the lag-filters. The overall system is stable for the whole range of $L_g$ variation around the initial value. This is because the design procedure considered the stability for the worst case with PM = 30 degrees when the grid inductance $L_g$ is nine times the initial value.

Fig. 10 shows the simulation results for the notch-filter case using the z-plane. The only parasitic resistances considered are $R$ and $R_g$, the extra line-inductance is purely inductive. The current-loop cannot be stable by the passive damping provided by these limited resistance values and active damping is required. The current-loop is stable for all the situations, but the system resonates for the initial inductance value of $L_g$. This is because the sampling frequency is very close (2.4 times) to the resonance frequency and the achieved damping is not sufficient. Active damping algorithms usually require that the sampling frequency be more than 3 times the resonance frequency [41]. After three cycles, an extra inductance of 9 times the initial inductance $L_g$ (2.25 mH) is connected and the current-loop remains stable, which demonstrates the robustness of the proposed new active damping procedure based on lag-filters. The current harmonic distortion improves considerably from THD=52% to THD=0.47% because of this extra inductance. Finally, after eight cycles the active damping mechanism is deactivated and the current-loop becomes unstable as expected.
Finally, it should be mentioned that unsuccessful attempts were made to use well-established procedures for active damping [41]. This remarks that the proposed procedures were successful in the face of difficult conditions to achieve stability with the LCL-filter based three-phase NPC inverter using the parameters shown in Table I.

VI. EXPERIMENTAL RESULTS

Fig. 11 shows a picture of the 100 kVA set-up used for the experiments. The three-level NPC grid-tie inverter is directly connected to a Regatron programmable three-phase power supply. The initial value of the grid inductance $L_g$ is 0.25 mH ($L_g = 0.5L$) and during the tests the grid inductance is increased up to 0.75 mH in a first step and up to 1.25 mH ($L_g = 2.5L$) in a second step. The grid inductance increments are sudden by opening the bypass breakers that short-circuit extra inductors.

Figs. 12-14 show the oscilloscope captures when the active damping procedure based on lag-filters is connected. Fig. 12 shows the grid voltage and the current waveforms after a 25 Amp step change in the current reference. The response is critically damped and consistent with the simulation shown in Fig. 6. The system behavior is stable but the current exhibits a large ripple because the switching frequency is very close (1.2 times) to the resonance frequency. The experiments are fully consistent with the simulations shown in Fig. 10. Fig. 13 shows the grid voltage and current waveforms for the sudden addition of extra line-inductance with a value twice the initial value of $L_g$ (0.5 mH). At the beginning of this experiment, the grid inductance $L_g$ has its initial value (0.25 mH). After adding the extra line-inductance, the grid inductance $L_g$ is three times its initial value (0.75 mH). The system remains stable and the current harmonics are reduced due to the filtering capability of the additional inductance. Finally, Fig. 14 shows the grid voltage and current waveforms for another sudden addition of extra line-inductance with a value twice the initial value of $L_g$ (0.5 mH). At the beginning of this experiment, the grid inductance is three times its initial value of $L_g$ (0.75 mH). After adding another extra line-inductance, the grid inductance $L_g$ is five times its initial value (1.25 mH). The system remains stable with a grid inductance $L_g$ of 5 times its initial value (1.25 mH), which demonstrates the robustness against line-inductance variations of the proposed novel active damping procedure based on lag-filters.

For both cases, experiments are fully consistent with theoretical derivations and simulations. The experiments present...
of this experiment, the grid inductance has its initial value \( L \). After adding another extra line-inductance, \( L \) inductance with a value twice the initial value of \( L \). For the ratio \( L_0/L = 1.6 \) as selected in [29], the THD achieved is 1.57%.

The results in the simulations and experiments for the notch-filter (using the z-transform matched rule) are very similar to those of the lag-filter. The root locus analysis in the z-plane shows that the system is stable for the whole range for \( L_0 \) varying from 80% to 1000%. In the simulation results, the systems initially resonated with THD=52% with the sampling frequency is very close (2.4 times) to the resonance frequency and finally it achieves a THD=0.47% after applying 9 times the initial inductance \( L_0 \). The system achieves a THD<5% for \( L_0/L = 0.8 \) \( L_0 =0.4 \text{ mH} \), 1.6 times the initial value of \( L_0 \). For the ratio \( L_0/L = 1.6 \) as selected in [29], the THD achieved is 1.53%. The main disadvantage of this the notch filter procedure is the necessity of knowing the resistance of the inductors.

VIII. Conclusion

This paper present a list of requirements for active damping in \( LCL \)-filter based medium-voltage grid-tie parallel inverters. The proposed solutions use filters in the voltage reference to the modulator. The presented approach allows selecting the bandwidth to be sacrificed for using active damping by calculating the Padé aproximant. The proposed tuning procedure for the notch-filter increases the robustness against the line-impedance variations using a simple design procedure that that requires to know the resistance of the inductors. The method of the lag-filters selects the phase margin for the worst case increase in the grid inductance to guarantee stability. Therefore, this proposed procedure constitutes an appropriate solution for this industrial application related to wind energy.

REFERENCES

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