Optimised GTO-GEO Transfer Using Low-Thrust Propulsion

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This paper proposes a global optimisation of the low-thrust transfers from GTO to GEO incorporating different types of perturbation. The trajectory transcription method makes use of an analytical solution of the perturbed Keplerian motion together with a simple direct collocation of the thrust arcs. The paper will show that low-thrust GTO to GEO transfers exhibit a number of local minima with a small but not negligible difference. The paper presents different strategies to explore the set of local minima and shows a number of locally optimal solutions.

Key Words: GTO-GEO transfer, electric propulsion, trajectory optimisation

1. Introduction

In this work an optimal electric propulsion transfer from Geostationary Transfer Orbit (GTO) to Geostationary Equatorial Orbit (GEO) is studied. Electric propulsion has been used since mid-1997 for the station keeping of Geostationary satellites. More recently, in March 2015, two Boeing all-electric satellites performed for the first time an electric-propelled orbit raising to the Geostationary Equatorial Orbit*. Low-thrust trajectories are indeed an efficient alternative to chemically propelled ones, since they have the potential to provide increased mass delivered to destination and smaller launch vehicles.

Several works in the literature have studied methods to optimise the transfer of spacecraft from GTO to GEO. Kluever uses a direct optimisation method to solve the GTO-GEO transfer. The weights to three optimal feedback control laws (for the variation of semimajor axis, eccentricity and inclination) are obtained as solution of a non-linear programming problem (NLP) in which the objective is the minimisation of the time of flight. The trajectory is propagated using orbital averaging and including perturbations from Earth shadows, oblateness and solar cell degradation. The same author also studied the GTO-GEO transfer with variable specific impulse. In this case the costates time histories are parametrised by linear interpolation and the design variables of the NLP are the nodal values of the costates. The objective is the minimisation of the fuel mass and a local optimisation method is used to solve the problem. Graham studied a direct method transcribed using GPOPS-II and solved using IPOPT, with analytical first and second derivatives computed by means of the software ADiGator. These methods, and the methods traditionally used to optimise low-thrust trajectories, are local methods. As such, they are able to find a solution, not the best solution. Moreover, the NLP methods require an initial guess that is not only hard to find but that also generally causes the optimiser to converge to an optimal trajectory close to the initial guess (that is rarely close to the global optimum). To overcome the limitation of local optimisation methods, effective global optimisation techniques are required. Global optimisation of low-thrust trajectory in the literature mainly focuses on the optimisation of interplanetary transfers.

In this work a global optimisation technique is applied to the GTO-GEO low-thrust transfer. The objective is the minimisation of the fuel consumption for the GTO-GEO transfer in a given time of flight. During the transfer the thrust is applied on two or four thrust arcs, two of which are centred at the perigee and apogee of the transfer orbits. The control parameters are the length of the thrust arcs and the elevation angle of the low-thrust vector. A population-based stochastic global optimisation algorithm, Multi-Population Adaptive Inflationary Differential Evolution Algorithm (MP-AIDEA), is used to globally explore the search space, and no user-defined initial guess is required to start the optimisation process. The model used for the motion of the spacecraft is an analytical propagator, which speeds up the optimisation process with respect to the use of a numerical one. The analytical propagator is based on non-singular equinoctial elements and includes low-thrust acceleration and perturbations due to Earth’s zonal harmonics J2, J3, J4, J5, atmospheric drag and third body gravitational perturbation from the Sun. Preliminary results show that many local minima exist for the solution of the minimum fuel low-thrust GTO-GEO problem. The paper starts with the description of the problem and of MP-AIDEA in Section 2. The optimal transfer is studied with no perturbations in Section 3, and with the perturbations due to Earth’s oblateness, drag and Sun gravitational attraction in Section 4. Section 5 concludes the paper.

2. The GTO-GEO global optimisation problem

This paper is concerned with the global optimisation of transfers from GTO to GEO with initial and final orbital parameters defined in Table 1. In Table 1, a, e, i, Ω, ω are the semimajor axis, eccentricity, inclination, right ascension of the ascending node and argument of the perigee. The nominal time of

<table>
<thead>
<tr>
<th>Table 1. GTO and GEO orbital elements.</th>
<th>a [km]</th>
<th>e</th>
<th>i [deg]</th>
<th>Ω [deg]</th>
<th>ω [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>GTO</td>
<td>24505</td>
<td>0.725</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>GEO</td>
<td>42165</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
flight for the transfer is $ToF = 225$ days. The spacecraft has initial mass $m_0 = 2000$ kg and engine characterised by thrust $T = 0.5$ N and specific impulse $I_p = 2000$ s. The thrust vector is defined by its magnitude $\epsilon$ and by its azimuth and elevation angles, $\alpha$ and $\beta$, in a radial-circumferential-normal reference frame (RCN):

$$a_R = \epsilon \cos \beta \sin \alpha, \quad a_C = \epsilon \cos \beta \cos \alpha, \quad a_N = \epsilon \sin \beta$$  \hspace{1cm} (1)

Initially the thrust is applied along two arcs per orbital revolution, at the perigee and apogee of each orbit.\textsuperscript{[7]} In the absence of perturbations and for the initial orbital elements given in Table 1, arcs centered at perigee and apogee represents ideal positions along the orbit to change semimajor axis, eccentricity and inclination. Considering the Gauss’ equations expressing the time variation of semimajor axis and eccentricity due to perturbations,\textsuperscript{[9]} it is possible to define the point of the orbit providing the maximum rate of change of $\alpha$ and $\beta$ by computing:

$$\frac{\partial \Delta a}{\partial \theta} = 0, \quad \frac{\partial \Delta e}{\partial \theta} = 0,$$  \hspace{1cm} (2)

where $\theta$ is the true anomaly. The previous equations give $\theta_0 = 0$ for the maximum rate of change of the semimajor axis and $\theta_0 = \pi$ for the maximum rate of change of the eccentricity, showing indeed that thrusting arcs centered at perigee and apogee provide the maximum instantaneous variation of semimajor axis and eccentricity. Following a similar analysis, the point of the orbit providing the maximum rate of change of $i$ is at true anomaly $\theta_i$ given by:\textsuperscript{[10]}

$$\sin (\theta_i + \omega) = -\epsilon \sin \omega$$  \hspace{1cm} (3)

Fig. 1 shows $\theta_i$ as a function of $\omega$ ranging from 0 to $2\pi$. Due to the arcsin term in Eq.(3), the optimal position to change $i$ is at $\theta_i$ and $\theta_i + \pi$. Fig. 1 shows that when $\omega = 0$ and no perturbations of the perigee arc is defined by the angle $\Delta L_p$ and the length of the apogee arc is $\Delta L_a$. The parameters defining the problem are $\Delta L_p$, $\Delta L_a$, and the elevation angle at perigee and apogee, $\beta_p$ and $\beta_a$. The azimuth angle on the two thrust arcs is not optimised as it follows one of the following four strategies: (1) Tangential thrust on both perigee and apogee thrust arc; (2) Tangential thrust at perigee, $\alpha = 0$ at apogee;\textsuperscript{[10]} (3) $\alpha = 0$ at perigee, tangential thrust at apogee; (4) $\alpha = 0$ on both perigee and apogee thrust arc.\textsuperscript{[9]} The control parameters are discretised during the transfer by considering four nodes to model the evolution of $\Delta L_p$, $\Delta L_a$, $\beta_p$ and $\beta_a$ from $t = 0$ to $t = ToF$. A linear interpolation is then used to define the value of the control parameters at any time during the transfer. The vector of parameters to optimise is defined, therefore, by 16 variables:

$$x = [\Delta L_{p1}, \Delta L_{p2}, \Delta L_{p3}, \Delta L_{p4}, \Delta L_{a1}, \Delta L_{a2}, \Delta L_{a3}, \Delta L_{a4}, \beta_{p3}, \beta_{p2}, \beta_{p1}, \beta_{a1}, \beta_{a2}, \beta_{a3}, \beta_{a4}]^T$$  \hspace{1cm} (4)

$\Delta L_{pi}, \Delta L_{ai}, \beta_{pi}, \beta_{ai}$ represents the value of the control parameters at node $i$. The state of the spacecraft is propagated using an averaged analytic propagator based on a first-order expansion of the perturbed equations of motion.\textsuperscript{[17]} The propagation is realised using non-singular equinoctial elements. The optimisation problem consists in the minimisation of the $\Delta V$ required to realise the transfer while constraining the final orbital elements at the end of the transfer to coincide with those of the GEO. The nonlinear constrained optimisation problem can be formulated as:

$$\min. f(x) = \Delta V$$  \hspace{1cm} s.t.  $g_j(x) \leq 0$  \hspace{0.5cm} $j = 1, \ldots, m$

$$h_k(x) = 0 \quad k = 1, \ldots, l$$

$$x^l_i \leq x_i \leq x^u_i \quad i = 1, \ldots, n$$  \hspace{1cm} (5)

The equality constraints $h(x)$ are:

$$h_1(x) = a(ToF) \left[1 - e(ToF)\right] - a_{GEO}$$

$$h_2(x) = a(ToF) \left[1 + e(ToF)\right] - a_{GEO}$$

$$h_3(x) = 10 \left[\sqrt{Q_1(ToF)^2 + Q_2(ToF)^2} - \tan \left(\frac{i_{GEO}}{2}\right)\right]^2$$  \hspace{1cm} (6)

where $a(ToF), e(ToF)$ are the semimajor axis and eccentricity at the end of the transfer and $Q_1(ToF)$ and $Q_2(ToF)$ are the third and fourth equinoctial elements at the end of the transfer. $a_{GEO}$ and $i_{GEO}$ are the semimajor axis and inclination of the target GEO (Table 1). The constraint on the final inclination is multiplied by 10 in order to match more precisely the final inclination of the GEO. The inequality constraints $g(x)$ are:

$$g_1(x) = R_0 - \min. [a(t)(1 - e(t))\]$$

$$g_2(x) = \max. \left[||\Delta L_p(t)|| + ||\Delta L_a(t)||\right] - 2\pi$$  \hspace{1cm} (7)

They impose that the minimum perigee radius during the transfer is higher than the Earth’s radius, $R_0$, and that the maximum sum of perigee and apogee thrust arc length is lower than $2\pi$. The lower and upper boundaries vectors are:

$$x^l_i = -2\pi, x^u_i = 2\pi, \quad i = 1, \ldots, 8$$

$$x^l_i = -\pi/2, x^u_i = \pi/2, \quad i = 9, \ldots, 16$$  \hspace{1cm} (8)
The stochastic global optimisation algorithm used in this study, MP-AIDEA, is not formulated to explicitly manage constraints. The constrained problem presented in Eq. (5) is therefore transformed into an unconstrained problem, applying a penalty method. The fitness function is expressed as a combination of the objective function and penalty constraints:

\[ f'(x) = f(x) + w_1 |g(x) > 0| \cdot |g(x)|^2 + w_2 |h(x)|^2 \]  

(9)

where \(w_1\) and \(w_2\) are appropriate weight coefficients. MP-AIDEA is a population-based evolutionary algorithm for solving single-objective global optimisation problems over continuous spaces. It combines adaptive Differential Evolution (DE), with the restarting procedure of Monotonic Basin Hopping (MBH). MP-AIDEA is able to automatically adapt the parameters of the DE and MBH during the optimisation. At the end of the DE a local search is run from the best individual of each population, and the local search algorithm solves the problem defined in Eq. 5. Using the restarting mechanism of MBH in combination with the DE, the populations are able to move, in a funnel structure, from one local minima to another, until the global minimum of the problem is located. MP-AIDEA implements also an approach to avoid multiple detection of the same local minima, by restarting the population in the entire work space when it falls within the basin of attraction of an already detected minimum. MP-AIDEA collects in an archive the local minima found during the exploration. It gives therefore the possibility to evaluate different possible solutions to the GTO-GEO transfer problem, each one corresponding to a different local minimum.

3. GTO-GEO transfer without perturbations

In this section the optimisation of the GTO-GEO transfer, without perturbations, is presented. At first, local solutions to the problem are found using different initial guesses, based on a pre-defined structure for the initial guess vector \(x_0\). The NLP problem presented in Eq. (5) is solved using MATLAB *fmincon-sqp*. MP-AIDEA is then used to globally explore the whole search space, using the fitness function defined in Eq. (9). For the local optimisation method, the vector of initial guess is:

\[ x_0 = [\Delta L_{p1,0} \Delta L_{p2,0} \Delta L_{p3,0} \Delta L_{p4,0} \Delta L_{a1,0} \Delta L_{a2,0} \Delta L_{a3,0} \Delta L_{a4,0} \beta_{p1,0} \beta_{p2,0} \beta_{p3,0} \beta_{p4,0} \beta_{a1,0} \beta_{a2,0} \beta_{a3,0} \beta_{a4,0}]^T \]  

(10)

The initial guess is constructed using values of \(\Delta L_{ai}\) linearly spaced from 0 to \(\Delta L_{a4,0}\):

\[ \Delta L_{ai,0} = \frac{\Delta L_{a4,0}}{3}(i-1), \quad i = 1, \ldots, 4 \]  

(11)

The initial guess of the length of the perigee thrust arcs corresponding to the first three nodes is:

\[ \Delta L_{p1,0} = \Delta L_{p2,0} = \Delta L_{p3,0} = 0 \]  

while \(\Delta L_{p4,0} \neq 0\). The initial guess for the elevation angles is:

\[ \beta_{p,i,0} = \beta_{a,i,0} = 0, \quad i = 1, \ldots, 4 \]  

(13)

Using Eq. (11), (12) and (13), the vector of initial guess \(x_0\) of Eq. (14) can be defined using only two parameters, \(\Delta L_{p4,0}\) and \(\Delta L_{a4,0}\):

\[ x_0 = [0 0 0 -\Delta L_{p4,0} 0 \frac{2\Delta L_{a4,0}}{3} \frac{\Delta L_{a4,0}}{3} 0 0 0 0 0 0 0 0 0]^T \]  

(14)

\(\Delta L_{ai} < 0\) correspond to thrust applied in the negative circumferential direction. Different values of \([\Delta L_{p4,0}]\) and \([\Delta L_{a4,0}]\) have been considered, in the range \([0, 180]\) deg, at interval of 10 degrees, resulting in a total of 100 local optimisation problems for the 4 thrusting strategies defined in Section 2. The results are shown in Fig. 2 to Fig. 5. Empty spaces in the plot represent conditions where the problem did not converge to a feasible solution using that initial guess. Results show that in all the cases higher \(\Delta V\) solutions are obtained at low values of \(\Delta L_{a4,0}\). For all the points that converged, the solutions are all different from zero. The results are shown in Fig. 2 to 5 and with values of the initial guesses of the elevation angles different from zero. The initial guess vector is now expressed.

![Fig. 2. \(\Delta V\) for different initial guesses of \(\Delta L_{a4,0}\) and \(\Delta L_{p4,0}\), Strategy 1.](image)

![Fig. 3. \(\Delta V\) for different initial guesses of \(\Delta L_{a4,0}\) and \(\Delta L_{p4,0}\), Strategy 2.](image)
as:

$$x_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} \Delta L_{p4,0}^{\text{min}\Delta V} \ 0 \ \frac{2\Delta L_{a4,0}^{\text{min}\Delta V}}{3} \ 0 \end{bmatrix} \begin{bmatrix} \beta_{p0} \ \beta_{p0} \ \beta_{p0} \ \beta_{p0} \ \beta_{p0} \ \frac{\beta_{a0}}{2} \ \beta_{a0} \ \frac{\beta_{a0}}{2} \end{bmatrix}^T$$

(15)

where $\Delta L_{p4,0}^{\text{min}\Delta V}$ and $\Delta L_{a4,0}^{\text{min}\Delta V}$ are the parameters corresponding to the minimum $\Delta V$ solution. Fig. 6 to 9 show the results when changing $\beta_{p0}$ and $\beta_{a0}$ from 0 to $\pi/2$. Results show that the effect of the initial guess of the elevation angle on the final solution is limited with respect to the effect of the length of the perigee and apogee thrust arcs. The minimum $\Delta V$ solutions are

**Fig. 4.** $\Delta V$ for different initial guesses of $\Delta L_{a4,0}$ and $\Delta L_{p4,0}$, Strategy 3.

**Fig. 5.** $\Delta V$ for different initial guesses of $\Delta L_{a4,0}$ and $\Delta L_{p4,0}$, Strategy 4.


<table>
<thead>
<tr>
<th>Table 2. Minimum $\Delta V$ solution for GTO-GEO transfer with no perturbations.</th>
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<tbody>
<tr>
<td>Strategy</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
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<tr>
<td>4</td>
</tr>
</tbody>
</table>

The results presented above are obtained solving a local minimisation problem and using a predefined specific expression for the initial guess. Each solution is therefore likely to be a local minima of the problem and might be strongly dependent on the choice of the initial guess. In the following, the results found using the global optimisation algorithm MP-AIDEA are presented. The local search in MP-AIDEA is performed with
MP-AIDEA is run with 1 population of 16 individuals (dimension of the problem) and for a total of $1.5 \times 10^5$ function evaluations; 25 independent runs are considered in order to obtain statistically significant results. The results are shown in Figs. 10 to 13, for the four considered thrusting strategies. For each strategy, the minimum $\Delta V$ for each one of the 25 runs of MP-AIDEA is represented. Three possible values of $w_1$ and $w_2$ are considered: 1, 10 and 100. The results of the local optimisation method (Table 2) are represented by the black lines. Results show that the local optimisation method outperforms MP-AIDEA when the initial guess vector is close to the solution of the problem (Fig. 10) while Fig. 13 shows a case in which the global search capabilities of MP-AIDEA are able to locate better solutions than those found by the local optimisation method. The minimum, maximum and mean values of the 25 runs, for each strategy, are reported in Table 3. The minimum $\Delta V$ solutions are obtained using Strategy 2 and $w = 1$ and Strategy 4 and $w = 100$ (results in bold in Table 3). The variation of orbital elements and the control parameters during the transfer for these two cases are shown in Fig. 14. $r_p$ and $r_a$ in Fig. 14 are the perigee and apogee radius. Fig. 15 shows the $x-y$ view of the trajectory for Strategy 4. To make the plot more readable, only few orbital revolutions are represented. The thrust arcs are represented by thick black lines.

Each one of the 25 runs of MP-AIDEA provides different solutions to the GTO-GEO transfer problem. As an example, Fig. 16 shows the solutions characterised by $\Delta V < 1.7$ km/s found by a single run of MP-AIDEA using Strategy 4 and $w = 100$. Each solution correspond to different control history and orbital elements variation to realise the GTO-GEO transfer. In particular in this case 233 solutions are found with $\Delta V < 1.7$ km/s, among which 120 with $\Delta V < 1.65$ km/s and 21 with $\Delta V < 1.6$ km/s. The orbital elements variation and control history of the 21 solutions with $\Delta V < 1.6$ km/s are shown in Fig. 17 and 18. A single run of MP-AIDEA can therefore find many local optima and many possible solutions to the problem.

## 4. GTO-GEO transfer with perturbations

The results presented in the previous section do not consider perturbations to the motion of the spacecraft. In this section the...
perturbations due to Earth’s potential, drag and third body are taken into account. Only strategy 4, that together with strategy 2 provided the best results in the case without perturbations, is considered.

4.1. Earth’s gravitation perturbations

This section starts presenting the results that justify the need to add two thrust arcs to the control profile when perturbations that change $\omega$ are present, as anticipated in Section 2. Let us consider the minimum $\Delta V$ solution of Section 3, given by MP-AIDEA using strategy 4 and $\omega_0 = 0$. This solution is used as initial guess for the local optimisation of the GTO-GEO transfer with the addition of the perturbation due to second zonal harmonic of the aspheric Earth’s potential, $J_2$. The orbital elements and control history are shown in Fig. 19 and 20. The cost of the transfer is $\Delta V = 1.7347$ km/s. Fig. 19 shows that...
there is an increase of inclination during the transfer from \( t = 25 \) days to \( t = 68 \) days. The reason for this behavior is explained in the following. At \( t = 25 \) days the argument of perigee (that changes both because of \( J_2 \) and because of the low-thrust acceleration) goes from \( \omega < \pi/2 \) to \( \omega > \pi/2 \). The Gauss’ equation for the time variation of the inclination depends on the term \( \sin \beta \cos(\omega + \theta) \); this means that in order to have a continuous reduction of inclination with thrust applied on perigee (\( \theta = 0 \)) and apogee (\( \theta = \pi \)) centered thrust arcs, \( \beta_a > 0 \) when \( \omega < \pi/2 \) and \( \beta_a < 0 \) when \( \omega > \pi/2 \). Therefore an instantaneous variation in the sign of \( \beta_a \) should take place at \( t = 25 \) days. Due to the type of control parametrisation and number of nodes used, the variation in the sign of \( \beta_a \) takes place however at \( t = 68 \) days, rather than 25 days (Fig. 20). This explains the increase in inclination from 25 to 68 days from the start of the transfer. At \( t = 68 \) days the inclination starts to decrease again. This behaviour shows that the control parametrisation used in the previous section requires some changes when considering the perturbation due to \( J_2 \), if period of increase of inclination are to be avoided during the transfer. In particular, since \( \omega \) changes during the transfer when perturbations are considered, the optimal point for the variation of \( i \) continuously changes during the transfer (Eq. 3). In order to allow for a reduction of inclination at any value of \( \omega \), two additional thrust arcs are added to the control parametrisation. They are characterised by length \( \Delta L_{pa} \) (thrust arc between perigee and apogee) and \( \Delta L_{ap} \) (thrust arc between apogee and perigee). The angular distance between any two arcs is constrained to be:

\[
\frac{2\pi - \Delta \beta_p - \Delta \beta_{pa} - \Delta \beta_{a} - \Delta \beta_{ap}}{4}
\]

(16)

The elevation angles on the two additional arcs is chosen such as to always cause a decrease of inclination, according to:

\[
\beta_{ap} = \beta_{pa} = \frac{\pi}{2} \text{sgn} (\cos(\omega + \theta))
\]

(17)

The control parameters are now 24, instead of 16:

\[
x = [\Delta L_{p1} \Delta L_{p2} \Delta L_{p3} \Delta L_{ap1} \Delta L_{ap2} \Delta L_{ap3} \Delta L_{ap4} \beta_{p1} \beta_{p2} \beta_{p3} \beta_{ap1} \beta_{ap2} \beta_{ap3} \beta_{ap4}]^T
\]

(18)

The disequality constraint \( g_2(x) \) in Eq. 7 is now formulated as:

\[
g_2(x) = \max \left( ||\Delta L_p(t)||, ||\Delta L_{ap}(t)||, ||\Delta L_{pa}(t)||, ||\Delta L_{ap}(t)|| - 2\pi \right)
\]

(19)

This new parametrisation of the control is used to solve the GTO-GEO transfer with perturbations due to \( J_2 \). As in the previous section, at first a local optimisation process is considered. The initial guess for \( \Delta L_{p1}, \Delta L_{ap}, \beta_p, \) and \( \beta_{ap} \) are the results of the best solution of the previous section. For \( \Delta L_{pa} \) and \( \Delta L_{ap} \) the value of their initial guess is taken in the range 0 to 180 deg. The results of the local optimisation of the problem, starting from different initial guess for \( \Delta L_{pa} \) and \( \Delta L_{ap} \), are shown in Fig. 21. The minimum \( \Delta V \) solution is represented in Fig. 22 and Fig. 23. The cost of the transfer is \( \Delta V = 1.6848 \) km/s, lower than the cost of 1.7347 km/s found with two thrust arcs, and the inclination decreases during the entire transfer. Results show that the additional thrust arcs have non-negligible semi-amplitude only in the last phase of the transfer (\( t > 150 \) days) when indeed the value of \( \omega \) approaches 90 deg, and therefore it is not efficient to change the inclination in the vicinity of the perigee and apogee of the orbit (Fig. 23). The solution has been validated by comparing it to the results of a numerical integration of the equations of motion using the control profile defined in Fig. 23. The comparison between numerical and analytic integration is shown in Fig. 24 and shows the good
agreement between the two models. The optimal solution to the
GTO-GEO transfer with $J_2$ perturbation is sought also using
MP-AIDEA. In order to facilitate convergence to the feasible
region, the search space is reduced with respect to the one pre-
sented in Eq. (8). The new boundaries for the search space
are $\Delta L_{\text{pp}} \in [-\pi/4, \pi/4]$, $\Delta L_{\text{ap}} \in [0, \pi/2]$, $\beta_{\text{pp}}, \beta_{\text{ap}} \in [-\pi/2, \pi/2]$
and $\Delta L_{\text{pp}}, \Delta L_{\text{ap}} \in [-\pi/4, \pi/4]$. MP-AIDEA is now run with 1
population of 24 individuals. The cost of the feasible solutions
found by one run of MP-AIDEA are shown in Fig. 25. By com-
paring Fig. 25 with Fig. 16 it is possible to see that the number
of solutions provided by a run of MP-AIDEA is now reduced
with respect to the case without perturbations. The minimum
cost solution found by MP-AIDEA is $\Delta V = 1.6588$ km/s, lower
than the value of 1.6848 km/s found by the local optimisation
(black line in Fig. 25). With the additions of perturbations, the

\[ \Delta V \text{ of feasible solution found by MP-AIDEA for GTO-GEO transfer with } J_2 \text{ perturbations and } \omega_0 = 0. \]

The initial value of $\omega$ of the GTO plays an important role. There-
fore the analysis presented above, valid for $\omega_0 = 0$ is realised
also for $\omega_0 = 178$ deg, the initial value of $\omega$ for the GTO of
the Ariane launcher \(^1\). Fig. 26 shows the results of the so-
lution of several local optimisation problem with initial guess
given by the solution without $J_2$ and $\omega_0 = 0$ and using values
for the initial guess of $\Delta L_{\text{pp}}$ and $\Delta L_{\text{ap}}$ in the range from 0 to $\pi$.
The minimum $\Delta V$ solution found by local optimisation is rep-

\[ \Delta V = \text{initial guess of } \Delta L_{\text{pp}}, \Delta L_{\text{ap}}, \omega_0 = 178 \text{ deg.} \]

solution is analysed in more detail to study the effect of additional perturbations: $J_2$, $J_3$, and $J_4$. No significant difference in $\Delta V$ is evident when considering these additional perturbations and the profile of the variation of the orbital elements remains approximately the same. In more detail, the $\Delta V$ and orbital elements at the end of the transfers are reported in Table 4.

### 4.2. Atmospheric Drag

In this subsection the effect of the atmospheric drag is analysed. The considered atmospheric model is a static exponential model with zero density of the atmosphere at altitude higher than 4000 km. No significant difference is measured when considering the perturbation due to the atmospheric drag for area to mass ratio of the spacecraft with typical values of $10^{-2}$ m$^2$/kg, as shown in Table 5. Table 5 shows the final orbital elements considering the optimal control profile defined in Subsection 4.1, and $J_2$, $J_3$, $J_4$, and the drag perturbation. Results show that, as expected, when using the control profile computed without the atmospheric drag, the addition of the atmospheric drag causes a reduction of the final semimajor axis. The reduction is however negligible and it is possible to state that the effect of the atmospheric drag is not significant for the considered GTO-GEO transfer.

#### Table 4. Final orbital element and $\Delta V$ - Effect of the Earth's perturbation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$J_4$</th>
<th>$J_3$, $J_4$, $J_5$</th>
<th>$J_2$, $J_3$, $J_4$, $J_5$, $J_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a [km]</td>
<td>42166.42</td>
<td>42166.34</td>
<td>42166.32</td>
<td>42166.32</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>1.41e-5</td>
<td>1.62e-5</td>
<td>1.68e-5</td>
<td>1.67e-5</td>
<td></td>
</tr>
<tr>
<td>i [deg]</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>$\Delta V$ [km/s]</td>
<td>1.6452</td>
<td>1.6452</td>
<td>1.6452</td>
<td>1.6452</td>
<td></td>
</tr>
</tbody>
</table>

#### Table 5. Final orbital elements and $\Delta V$ - Effect of the drag perturbation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$J_2$, $J_3$, $J_4$, $J_5$, $J_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a [km]</td>
<td>42166.42</td>
</tr>
<tr>
<td>e</td>
<td>1.41e-5</td>
</tr>
<tr>
<td>i [deg]</td>
<td>0.03</td>
</tr>
<tr>
<td>$\Delta V$ [km/s]</td>
<td>1.6452</td>
</tr>
</tbody>
</table>

#### Table 6. Initial $\Omega$ at different initial dates for the transfer

<table>
<thead>
<tr>
<th>Date</th>
<th>$\Omega_0$ [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 March</td>
<td>332.05</td>
</tr>
<tr>
<td>21 June</td>
<td>55.23</td>
</tr>
<tr>
<td>21 Sept.</td>
<td>148.92</td>
</tr>
<tr>
<td>21 Dec.</td>
<td>240.87</td>
</tr>
</tbody>
</table>
provide an initial guess to the solution. The addition of perturbations than a local optimisation method, without the need to explore the solution space and locate better solutions than a local optimisation method. This is illustrated in Fig. 32, where the black line represents the result of MP-AIDEA without the perturbation from the Sun (∆V = 1.6452 km/s). The final orbital elements at the end of the transfer and the ∆V of the best solution found for each one of the four considered dates are reported in Table 7.

Table 7. Final orbital elements and ∆V - Sun’s gravitational perturbation

<table>
<thead>
<tr>
<th>Day of the year</th>
<th>∆V [km/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 March</td>
<td>1.643</td>
</tr>
<tr>
<td>21 June</td>
<td>1.648</td>
</tr>
<tr>
<td>21 September</td>
<td>1.642</td>
</tr>
<tr>
<td>21 December</td>
<td>1.650</td>
</tr>
</tbody>
</table>

5. Conclusion

This paper presented the results of the global optimisation of the low-thrust transfer from GTO to GEO, including different types of perturbation. Results have shown that a global optimisation method can explore the solution space and locate better solutions than a local optimisation method, without the need to provide an initial guess to the solution. The addition of perturbations can cause differences in the results. In particular, the main difference with respect to the Keplerian case are caused by J₂; however also the Sun’s perturbation can cause small but non-negligible difference in the cost of the transfer.

References

8) Koppel, C. R.: Advantages of a continuous thrust strategy from a geosynchronous transfer orbit, using high specific impulse thrusters, 14th International Symposium on Space Flight Dynamics, 8-12 February 1999, Brazil.