ENERGY EFFICIENCY AS AN INSTRUMENT OF REGIONAL DEVELOPMENT POLICY? TRADING-OFF THE BENEFITS OF AN ECONOMIC STIMULUS AND ENERGY REBOUND EFFECTS

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Energy efficiency as an instrument of regional development policy? Trading-off the benefits of an economic stimulus and energy rebound effects

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Abstract

Previous studies show that improving efficiency in household energy use can stimulate a national economy through an increase and change in the pattern of the aggregate demand. However, this may impact competitiveness. Here we find that in an open region, interregional migration of workers may give additional momentum to the economic expansion, by relieving pressure on the real wage and the CPI. Furthermore, the stimulus will be further enhanced by the greater fiscal autonomy that Scotland is set shortly to enjoy. By considering a range of CGE simulation scenarios we show that there is a tension between the economic stimulus from energy efficiency and the scale of rebound effects. However, we also show that household energy efficiency increases do typically generate a “double dividend” of increased regional economic activity and a reduction in carbon emissions.

Key Words: energy efficiency, regional development policy, energy rebound, regional fiscal autonomy, general equilibrium

JEL codes: C68; D58; Q43; Q48; R28; R58
1 Introduction

In a recent report, the International Energy Agency (IEA, 2014) argues that increasing energy efficiency could deliver significant social and economic benefits that go beyond the traditional single objective of reducing energy demand. From an economic perspective, energy efficiency has been shown to positively impact on key macroeconomic indicators, such as employment, exports, and total output (Allan et al., 2007; Barker et al., 2007, 2009; Turner, 2009, 2013).

Computable General Equilibrium (CGE) models have often been used to investigate the economy-wide effects of energy efficiency improvements, including the ‘rebound effect’, because of their intrinsic multi-sectoral structure and whole economy characteristics (Gillingham and Rapson, 2016; Sorrell, 2007; Turner, 2013). Using CGE frameworks, studies focused on assessing rebound from energy efficiency increases in production have already underlined how a more efficient use of energy can deliver significant economic benefits. For example Broberg et al. (2015), Hanley et al. (2009), Turner (2009) and Yu et al. (2015) find that improving energy efficiency in production leads to a productivity-led expansion. The findings are quite intuitive, as in these studies energy is one of the production inputs, along with capital, labour and materials. This means that improving energy efficiency will deliver similar types of effects as improving capital or labour efficiency, although with some differences, given that energy is used in smaller proportions and is a produced rather than a primary input.

However, macroeconomic impacts of energy efficiency have also been observed when energy efficiency increases occur in household consumption. For example, Lecca et al. (2014) shows that a more efficient use of energy could lead to a reallocation of increased household expenditure towards non-energy sectors, thereby stimulating the economy through a shift in aggregate demand, but with some negative impacts on competitiveness and export demands.

The aim of this paper is to analyse the economy-wide impacts of increasing household energy efficiency in a regional context, accounting both for ‘costs’ of the rebound effect in energy use and for the potential benefits of energy efficiency. The Scottish Government, like many other regional and national governments, has multiple policy objectives, including sustainable economic growth, which itself reflects a positive weighting on both greater economic activity and lower carbon emissions. Accordingly, when assessing the impact of policies, including those relating to energy efficiency, it is appropriate to reflect these wider objectives. The focus should therefore not be exclusively on the impact on energy use. Indeed, we argue that household energy efficiency improvements can be regarded, in general, as an instrument of regional development policy, as well as a contributor to limiting carbon emissions.¹ Household

¹ The Scottish Government has recently designated improved energy efficiency within homes and non-domestic building stock as part of the National Infrastructure Priority. This reflects an increasing awareness of the role that energy efficiency might play in stimulating the regional economy.
energy efficiency improvements will typically yield a “double dividend”\(^2\) of increased regional economic activity and reduced energy use. Furthermore, the economic development effects of energy efficiency changes are permanent unlike the demand-side effects of any transitory increase in spending that may accompany the implementation of energy efficiency changes.

We use Scotland as a case study, comparing our analysis with Lecca et al. (2014), which focused on the UK national case. Here we use a purpose-built, regional energy-economy-environment Computable General Equilibrium (CGE) model of Scotland to analyse the impact of an improvement in household energy efficiency. Focusing on the case of Scotland allows us to highlight the implications of moving from a national to a regional context when analysing the system-wide impacts of household energy efficiency improvements. There are countervailing effects: the greater openness of regional economies leaves them more sensitive to induced changes in competitiveness; but the greater supply-side responsiveness of regional economies acts to limit the scale of any such changes. Overall, we find that household energy efficiency can be an effective instrument of regional development policy, and that it does indeed typically generate a double dividend.

The rest of the paper is organised as follows: In Section 2 we review the literature. In Section 3 we describe the CGE model used for this analysis. In Section 4 we illustrate the simulation scenarios. In Sections 5 and 6 we describe the results and discuss the main implications, in the context of the conventional fiscal arrangements for Scotland under which the budget constraint of the devolved Government does not vary with economic activity. In Section 7 we explore the impact of increased household energy efficiency for the case in which the Scottish Government enjoys a much greater degree of autonomy, as under the new fiscal arrangements that are currently in the process of being implemented. In Section 8 we draw conclusions.

2 Background

In a CGE framework, a number of authors have examined the economy-wide impacts of increased energy efficiency on the production/industrial side of the economy (e.g. Broberg et al., 2015; Grepperud and Ramussen, 2004; Glomsrød and Taoyuan, 2005; Koesler et al., 2016; Yu et al., 2015). Some studies have considered the case of UK and Scotland (see for instance Allan et al. 2007 and Turner 2009 for the UK; Anson and Turner 2009 and Hanley et al. 2009 for Scotland). However, all these contributors focus on efficiency improvements in production, and the economy-wide rebound effects (along with an expansionary impact on the economy) are driven by increased productivity and competitiveness.

To the best of our knowledge, few CGE studies focus on the economy-wide effects of increased household energy efficiency (Duarte et al., 2015; Dufournaud et al., 1994; Koesler, 2013; Lecca

\(^2\) The double dividend argument can be decomposed into a number of multiple benefits as intended by IEA (2014).
et al., 2014). Among the published works, Duarte et al. (2015) investigates different energy savings policies, including increased energy efficiency improvements, in Spain. However, this study is quite specific to the Spanish economy characterised by very different energy needs, compared to Scotland, and focusses mostly on the effectiveness of energy saving policies on CO₂ emissions.

Lecca et al. (2014) studies the economic impact of an across-the-board 5% improvement in the energy efficiency of a UK household. They illustrate the additional insights obtained in moving from partial to full general equilibrium analysis by calibrating models with different degrees of endogeneity on a common dataset. On this basis they show how it is possible to obtain a decomposition of economy-wide rebound effects into areas that may merit differential policy responses.

In Lecca et al. (2014) the general equilibrium analysis of energy efficiency is carried out in two stages. Firstly, the authors introduce an efficiency improvement to reflect an increase in the value of energy expressed in efficiency units, meaning that households can consume the original ‘pre-efficiency’ bundle of goods (energy and non-energy) but using less physical energy. This stimulates the wider economy through an increase in aggregate demand, because households would respond to the lower energy price (expressed in efficiency units) by substituting the consumption of non-energy goods for the consumption of energy goods. However, while in studies focused on industrial energy use, such as Allan et al. (2007) and Turner (2009), the economic expansion is driven by an increase in competitiveness, in Lecca et al. (2014) the demand-led growth puts upward pressure on consumption prices and so decreases competitiveness, partially crowding out exports.

Secondly, to understand how this loss in competitiveness may be avoided, Lecca et al. (2014) hypothesise that the energy efficiency improvement in household energy use is reflected in an overall decrease in the cost of living. They model this by simply adjusting the consumer price index (cpi) so that it is calculated to include the price of energy goods expressed in efficiency units and the price of non-energy goods. Thus, when energy efficiency improves, the cpi decreases, increasing competitiveness and putting downward pressure on the nominal wage.

In this paper, we build on the general equilibrium analysis of Lecca et al. (2014) but focus on a regional case study within the UK, using a single region CGE model of the Scottish economy. In order to emphasise the implications of moving from a national to a regional context, we initially replicate the type of analysis carried out in Lecca et al. (2014), but using a regional CGE model for Scotland³. Then, we extend this analysis by relaxing the assumption of a fixed working population imposed in Lecca et al. (2014) to consider the impacts of interregional migration in

³ The key differences between the national and the regional modelling contexts are explained in section 3.
response to differences in relative unemployment and wage rates. This provides another mechanism by which the competitiveness effects observed in the national case may be mitigated. Finally, we explore the implications for this analysis of enhanced fiscal autonomy in Scotland by exploring the consequences of assuming that the Scottish Government balances its own budget, thereby providing an additional source of stimulus where the economy is expanding since the additional tax revenues may be used either to increase regional public spending or reduce (devolved) tax rates.

3 The CGE model

To identify the general equilibrium impacts of energy efficiency we use the AMOS ENVl\(^4\) CGE model for Scotland. This model is based on the general AMOS CGE framework with forward-looking agents explained in Lecca et al. (2013) but extended to incorporate a more detailed structure of energy demand and supply (Lecca et al., 2014).

The regional focus of AMOS ENVl is reflected in two main characteristics. First, it does not impose a balance of payments constraint, to reflect the fact that regions do not possess a full range of fiscal and monetary policies, and receive transfers from the central Government (see Lecca et al., 2013, for a detailed discussion of this aspect). Second, it allows for flow migration, to reflect the free circulation of workers within the UK territory.

3.1 Consumption

Consumption is modelled to reflect the behaviour of a representative household that maximises its discounted intertemporal utility, subject to a lifetime wealth constraint. The solution of the household optimisation problem gives the optimal time path for consumption of the bundle of goods \(C_t\).

To capture information about household energy consumption, \(C_t\) is allocated within each period and between energy goods \(EC\) and non-energy goods \(NEC\) so that:

\[
C_t = \left[ \delta^E (\gamma EC_t)^{\frac{\varepsilon - 1}{\varepsilon}} + (1 - \delta^E) NEC_t^{\frac{\varepsilon - 1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}} - \frac{\varepsilon}{\varepsilon - 1}
\]

In (1) \(\varepsilon\) is the elasticity of substitution in consumption, and measures the ease with which consumers can substitute energy goods for non-energy goods; \(\delta \in (0,1)\) is the share parameter; and \(\gamma\) is the efficiency parameter of energy consumption. The consumption of energy is then divided into two composite goods: coal and refined oil; and electricity and gas. These in turn split into the four energy uses, refined oil, coal, electricity and gas, through a nested CES

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\(^4\) AMOS is the acronym of a micro-macro model of Scotland and it is the name of a CGE framework developed at the Fraser of Allander Institute, University of Strathclyde. ENVl indicated a version of this model developed for the analysis of energy/environmental impacts of a range of policies and other disturbances.
Moreover, we assume that the individual can consume goods produced both domestically and imported, where imports are combined with domestic goods under the Armington assumption of imperfect substitution (Armington, 1969).

3.2 Production and investment

The production structure reflects the classical KLEM nested CES production function, where capital and labour are combined together to form value added, and energy and materials are combined into intermediate inputs. The combination of intermediate inputs and value added forms gross output. Domestic and imported goods are combined under the Armington assumption (Armington, 1969).

The demand functions for capital and labour are obtained from the first order conditions of the CES production function. Following Hayashi (1982), the optimal time path of investment is derived from maximising the value of firms, $V_t$, subject to a capital accumulation function $K_t$, so that:

$$\text{Max} V_t \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \left[ \pi_t - I_t \left( 1 + g(x_t) \right) \right]$$

subject to $K_t = I_t - \delta K_t$

where $\pi_t$ is the firm’s profit, $I_t$ is private investment, $g(x_t)$ is the adjustment cost function, with $x_t = I_t / K_t$ and $\delta$ is depreciation rate. The solution of the problem gives the law of motion of the shadow price of capital, $\lambda_t$, and the adjusted Tobin’s q time path of investment (Hayashi, 1982).

3.3 The labour market, wage bargaining and migration

In this specification of the model, wages are determined within the region in an imperfect competition setting, according to the following wage curve:

$$\ln \left[ \frac{w_t}{cpi_t} \right] = \varphi - \epsilon \ln(u_t)$$

where the bargaining power of workers and hence the real consumption wage is negatively related to the rate of unemployment (Blanchflower and Oswald, 2009). In (3), $\frac{w_t}{cpi_t}$ is the real consumption wage, $\varphi$ is a parameter calibrated to the steady state, $\epsilon$ is the elasticity of the wage rate with respect to the rate of unemployment, $u$.

In the simulations below, the working population is initially assumed fixed, as in Lecca et al. (2014). However, as we have already noted, regions are much more open systems than

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5 See Appendix A.1 for a schematic representation of the consumption structure.
6 See Appendix A.2 for a schematic representation of the production structure.
nations, and the assumption of a fixed working population is likely to be inappropriate in a regional context. For this reason, we introduce the following migration function (Lecca et al., 2013):

\[
nim_t = \zeta - v^u [\ln(u_t) -\ln(\bar{u}^N)] + v^w [\ln(w_t/cp_i) - \ln(\bar{w}^N_t/cp^N_i)]
\] (4)

where \(nim_t\) is the instantaneous rate of net migration, \(\zeta\) is a parameter calibrated to ensure zero migration in the first period, and \(v^u\) and \(v^w\) are elasticities that measure the response to the differences in logs between regional and national unemployment and real wage rates. In Equation (4) net migration flows are positively related to the difference between the log of regional and national real wages and negatively related to the difference between the log of regional and national unemployment rates (Layard et al., 1991; Treyz et al., 1993). This means, for example, that when the regional real wage is higher than the national real wage and/or the regional unemployment rate is lower than its national counterpart, there will be net in-migration of workers to the region.

3.4 Modelling energy efficiency and the rebound effect

We define an increase in energy efficiency as any technological improvement that increases the energy services generated by each unit of physical energy (Lecca et al., 2014). This implies that the value of energy in efficiency units has risen. Consequently, the household can achieve the same level of utility by consuming the same amount of non-energy goods and services, but less physical energy.

For simplicity, we follow Koesler et al. (2016) and assume that the energy efficiency is given as a public good, with no cost of implementation for the household.\(^7\) This ensures comparability with the national case analysed by Lecca et al. (2014).

Following Lecca et al. (2014) we derive the economy-wide rebound effect in two stages. First, we consider the economy-wide rebound effect in the household sector \((R_C)\) as:

\[
R_C = \left[1 + \frac{\dot{E}_C}{\gamma}\right] \cdot 100
\] (5)

where \(\dot{E}_C\) measures the proportionate change in household energy consumption, which can be positive or negative, and \(\gamma\) measures the proportionate change in energy efficiency. Because we are analysing the household economy-wide rebound effect in a full general equilibrium

\(^7\) This assumption constitutes the focus of our future work.
system, \( \dot{E}_c \) results from a full range of economy-wide adjustments, not just the direct response to the change in the price of the energy service as efficiency increases.

Second, to identify the impact of the energy efficiency improvement on the whole economy (i.e. across all industries, household and domestic institutions) we derive the total rebound \( R \) as follows:

\[
R_T = \left[ 1 + \frac{\dot{E}_T}{\alpha \gamma} \right] \cdot 100
\]  

(6)

In this case, \( \dot{E}_T \) measures the proportionate change in the energy used in the whole economy, and \( \alpha \) is the initial household share of energy use in the base year.

It is important to notice that the term \( \frac{\dot{E}_r}{\alpha \gamma} \) can be expressed as:

\[
\frac{\dot{E}_r}{\alpha \gamma} = \frac{\Delta E_T}{\gamma E_C} = \frac{\Delta E_C + \Delta E_{P}}{\gamma E_C} = \frac{\dot{E}_C}{\gamma} + \frac{\Delta E_P}{\gamma E_C}
\]  

(7)

where \( \Delta \) represents absolute change and the subscript \( P \) indicates production. Substituting equations (5) and (7) into equation (6) gives:

\[
R_T = R_C + \frac{\Delta E_P}{\gamma E_C} \cdot 100
\]  

(8)

This shows that the total economy-wide rebound will be higher than the household economy-wide rebound if energy consumption in production increases as a result of the improvement in energy efficiency in the household sector.

To obtain additional insights from the nature of rebound, we decompose the total economy-wide rebound into the four energy uses included in the model as follows:

\[
R_{T,j} = \left[ 1 + \frac{\dot{E}_{T,j}}{\alpha_j \gamma} \right] \cdot 100
\]  

(9)

where the set \( j \) includes coal, gas, electricity and refined oil.

### 3.5 Data and calibration

To calibrate the model we follow a common procedure for dynamic CGE models (Adams and Higgs, 1990), which is to assume that the economy is initially in steady state equilibrium. The structural parameters of the model are derived from the 2009 Social Accounting Matrix (SAM) for Scotland (Emonts-Holley and Ross, 2014), which incorporates the 2009 Scottish Input-
Output tables. The Scottish SAM reports information about economic transactions between industries and other aggregate economic agents, namely the Scottish household, the Scottish Government, and corporate sectors, and accounts for imports and exports to the rest of the UK (RUK) and the rest of the world (ROW). For this paper, we aggregate the SAM to 21 industries\(^8\), including four energy sectors, gas, electricity, coal and refined oil.

The SAM constitutes the core dataset of the AMOS-ENVI model. However other parameters are required to inform the model, such as elasticities, and share parameters. These are either exogenously imposed, based on econometric estimation or best guesses, or determined endogenously through the calibration process.

To observe the adjustment of all the economic variables through time, simulations solve for 50 periods (years). We introduce a 5% costless, exogenous and permanent increase in the efficiency of energy used in household consumption. Following this initial ‘shock’, all the variables start to adjust over time until they reach a new steady state equilibrium. Results are reported for two conceptual periods: the short-run, where population and capital stocks are fixed, and the long-run, which corresponds to the new steady state equilibrium characterised by no further changes in sectoral capital stocks and population. We also report period by period adjustments.

4. **Simulation scenarios**

Our simulations reflect four main scenarios, summarised in Table 1. All of the simulations use the AMOS ENVI model, calibrated on Scottish data, as outlined in Section 3. Differences among the four Scenarios reflect the way the \(cpi\) is calculated and the degree of openness of the labour market.

\[
\begin{array}{|c|c|c|}
\hline
& \text{No Migration} & \text{Migration} \\
\hline
\text{Standard \(cpi\)} & \text{Scenario 1} & \text{Scenario 2} \\
\text{Adjusted \(cpi\)} & \text{Scenario 3} & \text{Scenario 4} \\
\hline
\end{array}
\]

**Scenario 1:** Here we use the variant of the AMOS-ENVI model that is most comparable to Lecca et al. (2014), in that the working population is assumed fixed. The \(cpi\) is also calculated in the standard way.

\(^8\) See Appendix B.1 for the full list of sectors included in the model
Scenario 2: In this scenario we repeat the simulations of Scenario 1, but incorporate endogenous migration, as in equation (4).

Scenario 3: Here we modify Scenario 1 by assuming that the energy efficiency improvement in the household sector is directly reflected in the wage determination process (equation 3), because the cpi effectively falls as a consequence of the improvement in energy efficiency (Lecca et al., 2014). This is implemented by adjusting the cpi to include the price of energy measured in efficiency units as follows:

\[ p_E^E = \frac{p_E}{1 + \gamma} < p_E \text{ for } \gamma > 0 \]  

so that

\[ cpi_x = cpi(p_{NE}, p_E^E) \]  

In (10) and (11) \( p_{NE} \) is the price of non-energy goods, \( p_E \) is the price of energy goods measured in natural units and \( p_E^E \) is the price of energy goods measured in efficiency units. When the price of energy in natural units is constant, an increase in efficiency decreases the price of energy in efficiency units, reducing therefore the cpi which directly affects the real wage as determined in equation (4). As in Scenario 1, the working population is fixed.

Scenario 4: We repeat the simulations carried out in Scenario 3, with the adjusted cpi (as in equations 10 and 11), but now allow for endogenous migration (equation 4).

To summarise, Scenarios 1 and 3 differ from one another in the way the cpi is calculated but they make the same fixed working population assumption as in Lecca et al (2014). Scenarios 2 and 4 repeat the same simulations as 1 and 3 but assume full flow migration.

As in Lecca et al. (2014) the short-run simulations for all scenarios are carried out using two alternative estimates of the elasticity of substitution between consumption of energy and non-energy goods, the short-run elasticity and the long-run elasticity. There are two main reasons for the use of two elasticities. First, there might be some degree of inertia in the adjustment of household consumption, which would be reflected in a lower response to an energy price change over the short period. Second, the energy efficiency improvement may come through an investment in durable goods. In this case, in order to access the efficiency improvement

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\(^9\) These are based on the most recent estimation carried out by Lecca et al. (2014) and are respectively 0.35 and 0.61
and adjustment of household capital stock would be necessary, and this is generally a long-run adjustment\textsuperscript{10}.

All of these simulations are based on the fiscal arrangements that existed prior to April 2016. Scotland is now in the process of moving to a significantly more devolved fiscal system: in particular, the Government’s budget will become dependent on Scottish income tax revenues, which vary directly with economic activity. In order to reflect this change we repeat the simulations from Scenario 1 in Section 6 below, but assume that the Scottish Government maintains a balanced budget so that any increased tax revenues resulting from the stimulus to economic activity generated by the increase in energy efficiency may be spent by the Government or used to reduce the rate of income tax.

5 Results

5.1 Scenario 1: the standard model with no migration

Table 2 summarises short-run (SR) and long-run (LR) results of simulations for Scenario 1. Note $\varepsilon$ is the elasticity of substitution in consumption between energy and non-energy goods. In the first column we report short-run results using the short-run elasticity of substitution ($\varepsilon_{SR}=0.35$). Following the energy efficiency improvement, household energy consumption decreases by 2.67%, while household consumption increases by 0.33%. The higher consumption puts upward pressure on the \textit{cpi}, making domestic products more expensive and reducing international competitiveness. On the other hand, this shift in demand stimulates investment in non-energy sectors, so that total investment increases by 0.14% and the output of non-energy producers rises by 0.7%. This impacts the labour market, where total employment increases by 0.06%, unemployment decreases by 0.25% and the real wage is 0.03% higher.

In the second column of Table 2 we report short-run results using the long-run elasticity (0.61). When the elasticity of substitution is low, consumers are more willing to substitute energy goods for non-energy goods. As the elasticity of substitution increases, the degree of substitutability decreases and consumers substitute less. In this case, there is less substitution away from energy to non-energy commodities, because the long-run elasticity is higher than the short-run, and this is reflected in a lower decrease in household energy consumption, -1.43%. Given the lower switch in consumption, the economic stimulus is also lower, reflecting the fact that, in the Scottish case, the expenditure in non-energy goods has a higher impact on the economy than the same spending on energy goods.

\textsuperscript{10}We plan to expand this aspect in the future work to analyse the case where the energy efficiency improvement is embedded in an investment in durable goods.
Long-run results are reported in the third column of Table 2. Scottish GDP increases by 0.11% relative to what it would have been without the efficiency improvement. The fall in household energy demands impacts energy demanded in production, which decreases by 0.22%. This is mostly due to the decreased activity in energy intensive energy suppliers. In fact, energy production and supply require lots of energy: when households demand less energy, less energy is supplied, and energy producers/suppliers reduce their energy use. For these reasons, the output of energy sectors decreases by 0.41%. Moreover, the initial decrease in demand for energy (as efficiency increases) causes a reduction in the return on capital in energy supply so that, over time, energy suppliers reduce their capacity. This is what Turner (2009) calls ‘the disinvestment’ effect.
This can be clearly seen in Figure 1 where we plot the shadow price of capital for the energy sectors and the replacement cost of capital. In the short-run the shadow price of capital of each sector drops below the replacement cost of capital, so that Tobin’s q is lower than 1 and therefore the cost of replacing the capital is higher than the value of the stock, and it is not profitable to invest. Over time, the price of energy rises again, allowing the shadow price of capital to recover and converge asymptotically to the replacement cost of capital, so that Tobin’s q again approaches unity. Because of the net contraction in industrial energy use, the overall long-run economy-wide rebound effect (50.08%), is smaller than the general equilibrium household rebound effect (70.33%).
Interesting insights can be obtained by disaggregating the rebound effects for each energy sector using Equation (9). In Figure 2 we plot household and economy-wide rebound effects disaggregated into coal, refined oil, electricity and gas. There is significant variation in the economy-wide rebound in the use of different types of energy, reflecting the different composition in the energy used in the production side of the economy. The economy-wide rebound in the use of electricity and gas is higher than the total economy-wide rebound, while in the case of refined oil rebound it is lower. There is a negative rebound in the use of coal, implying that the energy saved in this sector is higher than the expected savings. It is important to notice that household and firms do not usually consume coal directly, but rather they consume electricity produced by coal-fired power stations. When the demand for electricity drops, power stations cut the demand for coal, and this dramatically reduces the use of such fuel, explaining the negative rebound.

Results from Scenario 1 appear to be in line with findings in Lecca et al. (2014). However, given the higher degree of openness of the goods market of regions, exports decrease in Scotland by more than in the national case\textsuperscript{11}. However, the increase in household energy efficiency yields a “double dividend” of increased economic activity (and employment) and a reduction in total energy use across all simulations in Scenario 1.

5.2 Scenario 2: the standard model with migration

In this Scenario we repeat the simulations of Scenario 1, but include the migration function described by equation (4). Results for key variables are reported in Table 3. To facilitate the comparison with the no migration case, we add a fourth column reminding us of the long-run results from Scenario 1. Short-run results are quite close to the previous case, because there is no migration in the first period, therefore a comparison is not necessary.\textsuperscript{12}

In the long-run there is a higher increase in GDP (0.17%), reflecting the higher level of capital stock (0.17%) and employment (0.18%). The differences are driven by the effect of the net in-migration triggered by the initial drop in the unemployment rate and by the rise in the real wage. Following the energy efficiency improvement, workers start to migrate into the region in response to wage and unemployment differentials from the second period. This puts downward pressure on wages, and increases the unemployment rate according to the wage setting curve (equation 3). The dynamics of these variables can be seen in Figure 3 where we plot the time path of the real wage, unemployment, \textit{cpi} and exports.

\textsuperscript{11} In the UK case, exports decrease by 0.08% in the short run and 0.04% in the long run (Lecca et al., 2013)

\textsuperscript{12} Short-run results are not exactly the same as Scenario 1 as in this model we have forward-looking agents, therefore some of the effects of migration are anticipated.
The real wage falls and the unemployment rate increases until they both approach zero, when the labour market reaches its long-run equilibrium. Similarly, the \( \text{cpi} \) returns to its base year value, allowing exports to increase again until the original competitiveness is completely restored. This is a crucial result, because it shows that unlike in Scenario 1 and in Lecca et al. (2014), where the higher \( \text{cpi} \) crowds out exports, in a regional economy with free movement of workers and flow migration, the negative effect on international competitiveness disappears in the long-run, due to the effect of migration on prices.

The restored long-run competitiveness contributes additional momentum to the economic stimulus. This is reflected in a rise in output of non-energy sectors of 0.19%. But because these activities use energy as an input in production, the energy output drop is slightly less than in previous scenarios, likewise the decrease in total energy use is slightly less. On the other hand, household energy consumption decreases by 1.47%, which is quite close to the outcome in Scenario 1. This is because the lower real wage decreases the household’s labour income, partly mitigating the response in consumption. For this reason, only the calculated economy-wide rebound effect is higher (53.5%) while the household rebound is hardly affected.

The zero variation in prices over the long-run indicates the presence of a pure demand response to the introduction of the energy efficiency improvement, similar to what we would expect in an Input-Output modelling framework. (McGregor et al, 1996). The economic expansion observed in this Scenario is entirely demand-driven. Again, the increase in household energy efficiency
generates a double dividend, although here with a greater stimulus to economic activity and smaller fall in total energy use than in Scenario 1.

5.3 Scenario 3: the model with adjusted cpi and no migration

In Scenarios 1 and 2, the energy efficiency improvement is modelled so as to reflect a simple change in consumer’s taste, with the macroeconomic effects being driven by the change in consumption patterns.

Here we consider the case where the increase in household energy efficiency use is reflected in an overall reduction in the cost of living, by adjusting the cpi to include the price of energy calculated in efficiency units according to equations (10) and (11).

Key results for this case are summarised in Table 4. Unlike Scenario 1, where the cpi increases immediately and remains above the initial level for all 50 periods, and Scenario 2 where it returns to its base year value in the long-run, here the cpi decreases both in the short-run and in the long-run, given the lower price of energy in efficiency units. Consequently the nominal wage decreases by 0.16% in the short-run and by 0.22% in the long-run, but because of the lower cpi the real wage increases by 0.9% and 0.16%.
The lower price of goods produced domestically stimulates the demand for Scottish goods from the rest of the UK and the rest of the world, and although in the short-run exports fall by 0.5% (which is less than what we observed in Scenarios 1 and 2), in the long-run they increase by 0.16%. This difference is crucial in terms of comparison with the standard case, because it says that when the energy efficiency improvement is reflected in less pressure for higher wages, we have a long-run improvement in competitiveness, similar to Allan et al. (2007) and Turner (2009) which focus on industrial energy efficiency. It is also important to notice that, given the greater openness of the goods market of regions, the long-run increase in exports is significantly higher than that reported in Lecca et al. (2014).

The increase in competitiveness along with the switch in aggregate demand triggers a bigger economic stimulus that is reflected in most of the key macroeconomic indicators. For example, investment increases by 0.44% in the short-run and 0.32% in the long-run. Consequently, the increase in labour and capital used in production has a positive effect on output which increases by 0.12% in the short-run and by 0.33% in the long-run.\(^{13}\)

There is a higher demand for energy by industry sectors. Intuitively, when the production of goods and services increases, industry would consume more energy in the production process.

---

\(^{13}\) In Lecca et al. (2014) GDP increases by 0.1 in the short-run and 0.24 in the long-run.
However, in the household sector the decrease in energy consumption is in line with what was reported for Scenarios 1 and 2. For this reason, the household rebound is only around 0.5% higher than the standard no migration case. However, the economy-wide rebound is higher in Scenario 3, both in the short-run (31%) and in the long-run (63%), reflecting the higher use of energy for industrial purposes. This suggests that the bigger stimulus to economic activity observed in Scenario 3 results in overall a higher use of energy and calculated rebound effect, although there is still a double dividend in that economic activity rises while energy use falls.

5.4 Scenario 4: the case of migration and adjusted cpi

In the final case, we include both the adjusted cpi, equations (10) and (11), and the migration function, equation (4). Results from these simulations are reported in Table 5.

<table>
<thead>
<tr>
<th>Elasticity of substitution</th>
<th>$\varepsilon$ SR</th>
<th>$\varepsilon$ LR</th>
<th>$\varepsilon$ LR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time period</td>
<td>Short-run</td>
<td>Long-run</td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.12</td>
<td>0.11</td>
<td>0.53</td>
</tr>
<tr>
<td>Consumer Price Index</td>
<td>-0.27</td>
<td>-0.28</td>
<td>-0.49</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>-0.77</td>
<td>-0.73</td>
<td>0.00</td>
</tr>
<tr>
<td>Total Employment</td>
<td>0.19</td>
<td>0.18</td>
<td>0.54</td>
</tr>
<tr>
<td>Nominal Gross Wage</td>
<td>-0.18</td>
<td>-0.19</td>
<td>-0.49</td>
</tr>
<tr>
<td>Real Gross Wage</td>
<td>0.09</td>
<td>0.08</td>
<td>0.00</td>
</tr>
<tr>
<td>Households’ Consumption</td>
<td>0.22</td>
<td>0.22</td>
<td>0.53</td>
</tr>
<tr>
<td>Investments</td>
<td>0.46</td>
<td>0.47</td>
<td>0.50</td>
</tr>
<tr>
<td>Export</td>
<td>-0.03</td>
<td>-0.02</td>
<td>0.35</td>
</tr>
<tr>
<td>Non-Energy Output</td>
<td>0.14</td>
<td>0.13</td>
<td>0.51</td>
</tr>
<tr>
<td>Energy Output</td>
<td>-0.38</td>
<td>-0.18</td>
<td>-0.07</td>
</tr>
<tr>
<td>Energy Use</td>
<td>-0.88</td>
<td>-0.42</td>
<td>-0.26</td>
</tr>
<tr>
<td>Energy Demand in Production</td>
<td>-0.18</td>
<td>-0.06</td>
<td>0.10</td>
</tr>
<tr>
<td>Households’ Consumption of Energy</td>
<td>-2.79</td>
<td>-1.55</td>
<td>-1.27</td>
</tr>
<tr>
<td>Household Rebound</td>
<td>44.17</td>
<td>71.62</td>
<td>74.53</td>
</tr>
<tr>
<td>Economy-wide Rebound</td>
<td>28.38</td>
<td>65.36</td>
<td>78.59</td>
</tr>
</tbody>
</table>

In this case, we observe the greatest economic expansion, reflected in most of the macroeconomic indicators. GDP rises by 0.53% in the long-run, driven by a 0.5% increase in

---

14 Again, here we may argue that in fact there are multiple “dividends” or benefits from energy efficiency. First, energy efficiency reduces to some extent final energy demand. Second, it increases household income, reducing poverty and fuel poverty and stimulating the aggregate demand. Third, the demand stimulus has an impact on other sectors of the economy (multiple benefits). These are enhanced when the cpi is adjusted to reflect the reduction in prices of energy in efficiency units.
capital stock and 0.54% in employment. The latter is determined by the combined effects of migration and the adjusted cpi on the labour market.

In the short-run, unemployment decreases by 0.77%, and although the nominal wage falls by 0.18%, the real wage increases by 0.09%, thanks to the decrease in the cpi. This triggers interregional net in-migration. Similarly to Scenario 2, the real wage and unemployment rate start to adjust until they converge to their initial levels in the long-run. This is different from the adjusted cpi case with no migration, where in the absence of additional workers from abroad the unemployment rate drops by 1.48% in the long-run. However, in this case the cpi does not return to zero in the long-run, but it behaves as in Scenario 3, decreasing in the long-run by 0.49%.

The lower cpi encourages individuals to consume more. Household’s consumption increases by 0.22% in the short-run, and 0.53% in the long-run. Because goods produced in Scotland become cheaper for foreign buyers, there is a 0.35% increase in exports over the long-term, similar to Scenario 3.

The increased competitiveness, along with the shift in domestic aggregate demand, puts upward pressure on the demand for energy in all the productive sectors. In the long-run, energy output decreases by 0.07%, and the overall use of energy in the economy decreases by 0.26%, thanks to a drop in household energy consumption of 1.27%. However, industries raise their long-run energy demand, and unlike all the other scenarios there is an increase in industrial energy use (of 0.1%) in the long-run. This is the most interesting result of this Scenario because it underlines that under certain conditions, an increase in energy efficiency in the household sector may lead to an increase in industrial energy consumption.

![Figure 4: Long-run investment in the energy sectors](image)
In Figure 4 we plot long-run investment in gas, refined oil, coal and electricity in the four Scenarios. In the first three cases investments are negative in all the energy sectors due to the disinvestment effect described in Scenario 1 (Turner, 2009). However in Scenario 4 the contraction in investment is lower in gas, coal and electricity, but investment is positive in the oil sector, which is quite important in the Scottish economy.

Because energy used by industries increases in the long-run the long-run economy-wide rebound effect is higher (though marginally) than the household rebound effect exactly as we would expect given equation (8).

In Figure 5 we plot the household’s and economy-wide rebound effect disaggregated by energy sectors. The economy-wide rebound in oil and electricity is higher than the household rebound, reflecting the rise in the use of these fuels in industry. Unlike Scenario 1, where we observed a negative rebound in the oil sector (see Figure 2), in this case there is a positive 27.9% economy-wide rebound indicating a rise in the demand for such fuel, but there is again a “double dividend”.

6 Discussion: trading-off economic benefits and rebound

Results from the four Scenarios show that increasing household energy efficiency in Scotland by 5% would stimulate the Scottish economy. However, there is a clear trade-off between economic benefits and achieved energy savings, which varies across scenarios, depending on whether the efficiency improvement influences the cpi and the wage bargaining process, and whether there is migration.
Table 6 summarises the calculated long-run rebound and household rebound effects, and the long-run percentage change in GDP in the four cases. In Scenario 1, with the standard cpi and no migration, the economic expansion is triggered by a pure demand shock, which puts upward pressure on domestic prices, crowding out exports. In this case, the calculated household rebound effect is 70.33%, which reduces to 50.08% when the whole economy is considered, so that, overall, 50.08% of the 5% expected energy savings will be offset by increased energy demand. In this Scenario, GDP increases by 0.11%.

In Scenario 2, the efficiency change delivers again a pure demand shock, with no change in competitiveness in the long-run, further stimulating economic activity. This results in a greater increase in GDP of 0.17%. For this reason, while the household rebound is quite close to the level of Scenario 1, the overall rebound increases to 53.48%, reflecting a higher energy demand by industries.

In Scenario 3, where the cpi is adjusted to include the price of energy in efficiency units, but there is no migration, we observe an increase in competitiveness in the long-run and the type of stimulus is similar to the productivity-led growth observed in previous work focussed on energy efficiency in production (Allan et al., 2007; Turner, 2009). In this case, the household rebound effect is 71.07%, very close to Scenarios 1 and 2. However, given the stimulus to supply, industries demand more energy, delivering an overall rebound of 63%, and a 0.33% rise in GDP, which is greater than Scenarios 1 and 2.

Lastly, in Scenario 4, the combination of the adjusted cpi and migration would cause the largest supply side response, reproducing again the characteristics of a productivity-led stimulus, and triggering the greatest economic expansion. In fact, GDP rises by 0.53% and as we would expect, the economy-wide rebound is 78.6%, which is higher than the household’s rebound.

There is a clear trade-off between economic benefits and energy demand reduction, reflected in the fact that the higher the economic stimulus received from the more efficient use of energy, the higher the rebound effect. However, in none of these scenarios does the calculated rebound effect offset completely the expected energy reduction (i.e. there is no “backfire” effect), indicating that changes in household energy efficiency typically generate a “double dividend” of
an increase in economic activity and a reduction in energy use. Nonetheless, the stronger the economic stimulus, the smaller the reduction in energy use and the greater the extent of rebound.

7 Towards new fiscal powers for Scotland

In all the Scenarios above, we have treated Scotland as a regional economy that has no devolved taxes, which was the case until very recently. In these circumstances Government expenditure is entirely exogenous and tax revenues accrue to the central Government in Westminster.

However, with the gradual devolution of fiscal powers from UK to Scotland, this will be an increasingly inaccurate representation of the Scottish fiscal framework. Given that we are still in a transition period, here we illustrate the key principles by focussing on the simple case where the Scottish Government maintains a fixed government budget according to this simple relation:

\[ \text{GOVBAL}_t = GY_t - GEXP_t \]  \hspace{1cm} (12)

Equation (12) indicates that at each period the Government's budget GOVBAL is equal to Government income GY minus Government expenditure GEXP. In order to keep GOVBAL constant the Government can either increase/decrease its income by varying the rate of income tax or increase/decrease its current expenditure. We assume that whenever Government expenditure varies, the change is distributed across sectors, according to the baseline Government's expenditure shares.

To illustrate the implications of this assumption we repeat the simulations of Scenario 1, which reflects a 5% increase in household's energy efficiency assuming no interregional migration.

We explore 3 sub-scenarios, FIXGOV, FIXBAL, TAX. The FIXGOV Scenario replicates Scenario 1 by assuming fixed Government expenditure with tax revenues accruing to Westminster. In the FIXBAL case we assume that tax revenues are devolved and the Scottish Government maintains a given fiscal balance by varying public expenditure in response to any changes in tax revenues. In the TAX scenario we assume that the any stimulus to the economy, and to tax revenues, is used to reduce the income tax rate so as to maintain a fixed fiscal balance. FIXGOV results are reported in the first column of Table 7. The economic stimulus from the improved household's energy efficiency generates additional tax revenue for the Scottish Government. However, because expenditure is fixed and revenues accrue to the UK,

\[ \text{This is a simplifies version of Equation C56 in Appendix C.} \]
not the Scottish Government, the Scottish Government’s fiscal balance increases both in the short-run and in the long-run.

Table 7: Comparing impacts of a 5% increase in household energy efficiency under different fiscal regimes

<table>
<thead>
<tr>
<th></th>
<th>FIXGOV</th>
<th></th>
<th>FIXBAL</th>
<th></th>
<th>TAX</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SR</td>
<td>LR</td>
<td>SR</td>
<td>LR</td>
<td>SR</td>
<td>LR</td>
</tr>
<tr>
<td>GDP</td>
<td>0.04</td>
<td>0.11</td>
<td>0.05</td>
<td>0.14</td>
<td>0.05</td>
<td>0.19</td>
</tr>
<tr>
<td>CPI</td>
<td>0.09</td>
<td>0.04</td>
<td>0.11</td>
<td>0.05</td>
<td>0.11</td>
<td>0.02</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>-0.25</td>
<td>-0.45</td>
<td>-0.32</td>
<td>-0.61</td>
<td>-0.34</td>
<td>-0.76</td>
</tr>
<tr>
<td>Total employment</td>
<td>0.06</td>
<td>0.11</td>
<td>0.08</td>
<td>0.15</td>
<td>0.08</td>
<td>0.19</td>
</tr>
<tr>
<td>Nominal Gross Wage</td>
<td>0.12</td>
<td>0.09</td>
<td>0.14</td>
<td>0.12</td>
<td>0.12</td>
<td>0.04</td>
</tr>
<tr>
<td>Real Gross Wage</td>
<td>0.03</td>
<td>0.05</td>
<td>0.04</td>
<td>0.07</td>
<td>0.04</td>
<td>0.09</td>
</tr>
<tr>
<td>Household’s Consumption</td>
<td>0.33</td>
<td>0.40</td>
<td>0.37</td>
<td>0.44</td>
<td>0.41</td>
<td>0.52</td>
</tr>
<tr>
<td>Investment</td>
<td>0.12</td>
<td>0.11</td>
<td>0.15</td>
<td>0.13</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
<td>Exports</td>
<td>-0.13</td>
<td>-0.06</td>
<td>-0.15</td>
<td>-0.08</td>
<td>-0.15</td>
<td>-0.03</td>
</tr>
<tr>
<td>Non-Energy Output</td>
<td>0.07</td>
<td>0.14</td>
<td>0.09</td>
<td>0.17</td>
<td>0.09</td>
<td>0.21</td>
</tr>
<tr>
<td>Energy Output</td>
<td>-0.41</td>
<td>-0.46</td>
<td>-0.41</td>
<td>-0.46</td>
<td>-0.40</td>
<td>-0.39</td>
</tr>
<tr>
<td>Energy Use</td>
<td>-0.88</td>
<td>-0.61</td>
<td>-0.86</td>
<td>-0.59</td>
<td>-0.85</td>
<td>-0.52</td>
</tr>
<tr>
<td>Energy Demand by Industries</td>
<td>-0.22</td>
<td>-0.30</td>
<td>-0.22</td>
<td>-0.28</td>
<td>-0.21</td>
<td>-0.22</td>
</tr>
<tr>
<td>Household’s Energy Consumption</td>
<td>-2.67</td>
<td>-1.48</td>
<td>-2.63</td>
<td>-1.44</td>
<td>-2.60</td>
<td>-1.37</td>
</tr>
<tr>
<td>Government Expenditure</td>
<td>-</td>
<td>-</td>
<td>0.06</td>
<td>0.16</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Government Balance</td>
<td>56.7</td>
<td>124.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Income Tax</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.10</td>
<td>-0.26</td>
</tr>
<tr>
<td>Household Rebound</td>
<td>46.57</td>
<td>70.33</td>
<td>47.32</td>
<td>71.10</td>
<td>48.08</td>
<td>72.66</td>
</tr>
<tr>
<td>Economy Wide Rebound</td>
<td>28.40</td>
<td>50.09</td>
<td>29.53</td>
<td>51.80</td>
<td>30.82</td>
<td>57.40</td>
</tr>
<tr>
<td>Energy productivity</td>
<td>0.79</td>
<td>0.81</td>
<td>0.80</td>
<td>0.82</td>
<td>0.79</td>
<td>0.78</td>
</tr>
</tbody>
</table>

In the FIXBAL case, the additional income is used to increase the Scottish Government’s current expenditure by 0.06% in the short-run and 0.16% in the long-run. The additional resources are now recycled into the economic system under the form of additional demand, further stimulating the economy. For this reason GDP increases by more than in the FIXGOV case, both in the short-run (by 0.05%) and in the long-run (by 0.14%). Similarly we observe a greater increase in employment, investment and output from industries. The additional Government spending puts additional pressure on domestic prices, further reducing exports. Consistently with what we observed in the other Scenarios of this paper, the greater economic expansion is also associated with bigger rebound effects.

Finally, in the TAX case, the results of which are reported in the third column of Table 7, the Government uses the additional resources to reduce the income tax rate. In this case we have a simultaneous demand and supply stimulus.
Firstly, tax reduction increases household’s disposable income so that consumption rises by 0.41% in the short-run and 0.51% in the long-run. Secondly, the reduced taxation increases the post tax real consumption wage, so that there is downward pressure on wage bargaining, reducing the price of labour and stimulating employment and production. The long-run nominal wage increases by 0.04% while it was 0.09% in the standard case. However, the real wage increases by 0.09% which is more that the FIXGOV and FIXBAL scenarios.

Because production is stimulated by the lower price of labour, industries produce more output, increasing also the use of other inputs, including energy. For this reason, the economy wide rebound is substantially higher than in the FIXGOV case, especially in the long-run (57.4%).

8 Conclusions

The simulation results reported in this paper leads us to five general conclusions.

First, increasing energy efficiency in Scottish households stimulates the regional economy. Increases in household energy efficiency do in fact act as a regional development policy. However, the scale and nature of the stimulus differs depending on the precise specification of the shock. The key issue here is whether the cpi is adjusted to reflect the lower price of an efficiency unit of energy. If the cpi is not adjusted the stimulus to the economy from the increase in household energy efficiency takes the form of a pure demand shock; if the adjusted cpi is relevant there is a simultaneous demand and supply side stimulus.

Second, moving from a national to a regional context, in particular by opening the labour market to migration, typically results in a greater economic stimulus. Even if migration is insufficient to fully restore initial wage and unemployment rates, the direction of the impact would be the same: the presence of migration reinforces the impact of any demand or supply side stimulus on the economy.

Third, the stimulus to household energy efficiency always reduces energy use. So household energy efficiency increases typically deliver a “double dividend” of reductions in energy demands (and emissions) and increases in economic activity. However, when the economic expansion is greater, the difference between potential energy savings and actual energy savings (rebound effects) is also higher, indicating a trade-off between actual energy savings and economic benefits. Energy efficiency stimuli do help with the achievement of energy or emission targets, but the extent to which they do so is generally inversely related to the scale of the associated economic expansion.

Fourth, greater regional fiscal autonomy will reinforce the economic stimulus, since in this case increases in regional economic activity stimulate the regional Government’s tax revenues, which can be used either to increase public spending, or to reduce Scottish tax rates. However, greater autonomy therefore also implies that the extent of energy saving will be reduced. This
is significant given that Scotland is in the process of acquiring a substantially enhanced degree of fiscal autonomy.

Finally, the drivers of the rebound effect are also the drivers of the economic stimulus. Further investigations should explore ways to minimise the magnitude of the rebound effect, without sacrificing the gains in terms of economic welfare.
References


Dufournaud, C.M., Quinn, J.T., and Harrington, J.J. (1994). An Applied General Equilibrium (AGE) analysis of a policy designed to reduce the household consumption of wood in the Sudan.


Appendix A

The structure of consumption and production

**Figure A.1: The Structure of Consumption**

```
Total consumption  
|               | Coal and Oil     | Coal |
| Non-Energy     | Energy           | Oil  |
|                | Electricity and Gas | Electricity |
```

**Figure A.2: The Structure of Production**

```
Gross Output
<table>
<thead>
<tr>
<th>Intermediary inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>UK</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Scotland</td>
</tr>
</tbody>
</table>
```

```
Appendix B

Industries included in the AMOS ENVI model

Table B.1: The industrial disaggregation of the AMOS ENVI 21- sectors model

<table>
<thead>
<tr>
<th>Sector’s name</th>
<th>Original sector from the 104 Scot IO table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, forestry and logging</td>
<td>1+2+3</td>
</tr>
<tr>
<td>Sea fishing and fish farming</td>
<td>4+5</td>
</tr>
<tr>
<td>Mining and extraction</td>
<td>7+8+9</td>
</tr>
<tr>
<td>Food, drink and tobacco</td>
<td>10 to 20</td>
</tr>
<tr>
<td>Textiles and clothing</td>
<td>21+22+23</td>
</tr>
<tr>
<td>Mfr Chemicals etc</td>
<td>28 to 35</td>
</tr>
<tr>
<td>Metal and non-metal goods</td>
<td>36+37+38</td>
</tr>
<tr>
<td>Transport and other machinery</td>
<td>39+40+41+42+43</td>
</tr>
<tr>
<td>Other manufacturing</td>
<td>24+25+26+44+45+46</td>
</tr>
<tr>
<td>Water, sewerage and waste</td>
<td>49+50+51</td>
</tr>
<tr>
<td>Construction</td>
<td>52+53+54</td>
</tr>
<tr>
<td>Distribution</td>
<td>55+56+57+64+65</td>
</tr>
<tr>
<td>Transport</td>
<td>58 to 63</td>
</tr>
<tr>
<td>Communications, finance and business</td>
<td>66 to 81+83 to 91</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>82</td>
</tr>
<tr>
<td>Education</td>
<td>93</td>
</tr>
<tr>
<td>Public and other services</td>
<td>92+94 to 104</td>
</tr>
<tr>
<td>Coal extraction</td>
<td>6</td>
</tr>
<tr>
<td>Oil (refining and distribution)</td>
<td>27</td>
</tr>
<tr>
<td>Gas</td>
<td>47</td>
</tr>
<tr>
<td>Electricity</td>
<td>48</td>
</tr>
</tbody>
</table>
Appendix C  The mathematical presentation of the AMOS-ENVI model

Prices

\[ PM_{i,t} = \overline{PM}_i \]  \hspace{1cm} (C.1)

\[ PE_{i,t} = \overline{PE}_i \]  \hspace{1cm} (C.2)

\[ PQ_{1,T} = \frac{PR_{i,t} \cdot Ri, t + PM_{i,t} \cdot Mi, t}{Ri, t + Mi, t} \]  \hspace{1cm} (C.3)

\[ PIR_{1,T} = \frac{\sum_i VR_{i,j,t} \cdot PR_{j,t} + \sum_i VI_{i,j,t} \cdot \overline{PI}_{j,t}}{\sum_i V1R_{i,j,t}} \]  \hspace{1cm} (C.4)

\[ PY_{j,t} \cdot a_{j} \gamma = \left( PR_{j,t} \cdot (1 - btax_j, sub_j, dep_j) - \sum_i a_{i,j} PQ_{j,t} \right) \]  \hspace{1cm} (C.5)

\[ UCK_t = PK_t \cdot (r + \delta) \]  \hspace{1cm} (C.6)

\[ PC_{t}^{1-\sigma^C} = \sum_j \delta_{j}^f \cdot PQ_{1-\sigma^C} \]  \hspace{1cm} (C.7)

\[ PG_{t}^{1-\sigma^G} = \sum_j \delta_{j}^g \cdot PQ_{1-\sigma^G} \]  \hspace{1cm} (C.8)

\[ PNE_t = \frac{\sum_z PQ_{z,t} \cdot \overline{V}_z}{\sum_z PQ_{z} \cdot \overline{V}_z} \]  \hspace{1cm} (C.9)

\[ PE_t = \frac{\sum_E PQ_{E,t} \cdot \overline{V}_E}{\sum_E PQ_{E} \cdot \overline{V}_E} \]  \hspace{1cm} (C.10)

\[ w_{t}^b = \frac{w_{t}}{1 + \tau_t} \]  \hspace{1cm} (C.11)
\[
\ln \left[ \frac{w_t}{cpi_t} \right] = \varphi - \epsilon \ln(u_t) \tag{C.12}
\]

\[
nim_t = \zeta - v^w \left[ \ln(u_t) - \ln(\bar{u}^N) \right] + v^w \left[ \ln(w_t/cpi_t) - \ln(\bar{w}^N_t/cpi^N_t) \right] \tag{C.13}
\]

\[
rk_{j,t} = PY_{j,t} \cdot \delta^k \cdot A^V_{\theta_j} \cdot \left( \frac{Y_{j,t}}{K_{j,t}} \right)^{1-\theta_j} \tag{C.14}
\]

\[
P_{k_t} = \frac{\sum_j PY_{j,t} \cdot \sum_i KM_{i,j}}{\sum_{i,j} KM_{i,j}} \tag{C.15}
\]

Production technology

\[
X_{i,t} = A^X_i \cdot \left[ \delta^X_i \cdot Y_{i,t}^{\rho^X_i} + (1 - \delta^V_i) \cdot V^{\rho^X_i} i_t \right]^{1\rho^X_i} \tag{C.16}
\]

\[
Y_{j,t} = \left( A^{\rho^Y_i} \delta^Y_i \cdot \frac{PQ_{j,t}}{PY_{j,t}} \right)^{1\rho^Y_j} \cdot X_{i,t} \tag{C.17}
\]

\[
V_{j,t} = \left( A^{\rho^V_i} (1 - \delta^V_i) \cdot \frac{PQ_{j,t}}{PV_{j,t}} \right)^{1\rho^V_j} \cdot X_{i,t} \tag{C.18}
\]

\[
v_{i,t} = A^v_i \cdot \left[ \delta^v_i \cdot E_{i,t}^{\rho^v_i} + (1 - \delta^E_i) \cdot NE^{\rho^v_i} i_t \right]^{1\rho^v_i} \tag{C.19}
\]

\[
\frac{E_{j,t}}{E_{j,t}} = \left[ \left( \frac{\delta^E_j}{1 - \delta^E_j} \right) \cdot \left( \frac{PNE_{j,t}}{PE_{j,t}} \right) \right]^{1\rho^E_j} \tag{C.20}
\]

\[
VV_{ze,j,t} = \left( A^{\rho^E_j} (1 - \delta^E N_i) \cdot \frac{PNE_{j,t}}{PQ_{E,j,t}} \right)^{1\rho^E_j} \cdot E_{i,t} \tag{C.21}
\]

\[
Y_{i,t} = A^Y_i \cdot \left[ \delta^Y_i \cdot K_{i,t}^{\rho^Y_i} + \delta^l_i \cdot L^{\rho^Y_i} i_t \right]^{1\rho^Y_i} \tag{C.22}
\]
\[ L_{j,t} = \left( A\rho_j \delta_l^j \cdot \frac{PY_{jt}}{w_t} \right)^{\frac{1}{1-\rho_j^l}} \cdot Y_{j,t} \]  

(C.23)

Trade

\[ VV_{i,j,t} = Y_{i,v} \cdot \left[ \delta_{i,v}^m \cdot VM_{i,t}^A + (1 - \delta_{i,v}^m) \cdot VIR_{i,t}^A \right]^{\frac{1}{\rho_i^V}} \]  

(C.24)

\[ \frac{VM_{i,j,t}}{VIR_{i,j,t}} = \left[ \left( \frac{\delta_{i,j}^m}{1 - \delta_{i,j}^m} \right) \cdot \left( \frac{PIR_{i,t}}{PM_{i,t}} \right) \right]^{1 - \rho_i^V} \]  

(C.25)

\[ VIR_{i,j,t} = Y_{i,v} \cdot \left[ \delta_{i,v}^m \cdot VI_{i,t}^A + (1 - \delta_{i,v}^m) \cdot VM_{i,t}^A \right]^{\frac{1}{\rho_i^V}} \]  

(C.26)

\[ \frac{VR_{i,j,t}}{VI_{i,j,t}} = \left[ \left( \frac{\delta_{i,j}^v}{1 - \delta_{i,j}^v} \right) \cdot \left( \frac{PI_{i,t}}{PR_{i,t}} \right) \right]^{1 - \rho_i^V} \]  

(C.27)

\[ E_{i,t} = \bar{E}_t \cdot \left( \frac{PE_{i,t}}{PQ_{i,t}} \right)^{\rho_i^E} \]  

(C.28)

Regional demand

\[ R_{i,t} = \sum_i VR_{i,j,t} + \sum_i QHR_{i,h,t} + QVR_{i,t} + QGR_{i,t} \]  

(C.29)

Total absorption equation

\[ X_{i,t} + M_{i,t} = \sum_i VV_{i,j,t} + \sum_i QH_{i,h,t} + QV_{i,t} + QG_{i,t} + E_{i,t} \]  

(C.30)

Households and other domestic institutions

\[ U^t(c_t) = \sum_{i=1}^{T-t} (1 + \rho)^{-t} C_{i-\sigma} - \frac{1}{1 - \sigma} \]  

(C.31)

\[ \frac{C_t}{C_{t+1}} = \left[ \frac{PC_t \cdot (1 + \rho)}{PC_{t+1} \cdot (1 + r)} \right]^{-\frac{1}{2}} \]  

(C.32)
\[ W_t = NFW_t + FW_t \quad (C.33) \]

\[ NFW_t(1 + r) = NFW_{t+1} + (1 - \tau_t)L_t^a(1 - u_t)w_t + Trf_t \quad (C.34) \]

\[ FW_t(1 + r) = FW_{t+1} + \Pi_t + S_t \quad (C.35) \]

\[ Trf_t = P_{ct} \cdot Trf \quad (C.36) \]

\[ S_t = mps \cdot [(1 - \tau_t)L_t^a(1 - u_t)w_t + Trf_t] \quad (C.37) \]

\[ \Pi = d^h \cdot \sum_i r_{ki,t}K_{ki,t} \quad (C.38) \]

\[ C_t = [\delta^E(\gamma EC_t)^{\rho_e} + (1 - \delta^E)NEC_t^{\rho_e}]^{-\frac{1}{\rho_e}} \quad (C.39) \]

\[ Ec_t = \left( \gamma^{\rho_e,\delta^E} \cdot \frac{P_{ct}}{P_{Et}} \right)^{\frac{1}{1-\rho_e}} \cdot C_t \quad (C.40) \]

\[ Ec_t = [\delta^{co}CO_t^{\rho_o} + (1 - \delta^{co})EG_t^{\rho_o}]^{\frac{1}{\rho_o}} \quad (C.41) \]

\[ \frac{CO_t}{EGc_t} = \left[ \left( \frac{\delta^{co}}{1 - \delta^{co}} \right) \cdot \left( \frac{PEG_t}{PCO_t} \right) \right]^{\frac{1}{1-\rho_o}} \quad (C.42) \]

\[ CO_t = [\delta^{cl}CL_t^{\rho_o} + (1 - \delta^{co})OIL_t^{\rho_o}]^{\frac{1}{\rho_o}} \quad (C.43) \]

\[ \frac{CL_t}{OIL_t} = \left[ \left( \frac{\delta^{cl}}{1 - \delta^{cl}} \right) \cdot \left( \frac{PQ_{oil,t}}{PQ_{coal,t}} \right) \right]^{\frac{1}{1-\rho_o}} \quad (C.44) \]
\[
QH_{z,t} = \left( \delta^{i} f^{c}_{i} \cdot \frac{P_{c_{i}}}{PQ_{z,t}} \right)^{\gamma_{ho_{i}}^{c}} \cdot NE_{c_{i}} \quad (C.45)
\]

\[
EG_{t} = \left[ \delta^{Ele} Ele^{\rho_{t}} + (1 - \delta^{el}) \right] \cdot \frac{1}{rh_{t}} \quad (C.46)
\]

\[
\frac{Ele_{t}}{GAS_{t}} = \left[ \left( \frac{\delta^{GAS}}{1 - \delta^{GAS}} \right) \cdot \left( \frac{PQ_{GAS,t}}{PQ_{Ele,t}} \right) \right]^{\frac{1}{1 - \rho^{\sigma}}} \quad (C.47)
\]

\[
QH_{ele,t} = EC_{t} \quad (C.48)
\]

\[
QH_{GAS,t} = GAS_{t} \quad (C.49)
\]

\[
QH_{Coal,t} = CL_{t} \quad (C.50)
\]

\[
QH_{OIL,t} = OIL_{t} \quad (C.51)
\]

\[
QH_{I,t} = \gamma_{i}^{f} \left[ \delta^{hi}_{i} QHIR_{i,t}^{\rho_{i}} + (1 - \delta^{hm}_{i})QHM_{i,t}^{\rho_{i}} \right]^{rac{1}{\rho_{i}}} \quad (C.52)
\]

\[
\frac{QHIR_{i,t}}{QHM_{i,t}} = \left[ \left( \frac{\delta^{hi}_{i}}{1 - \delta^{hm}_{i}} \right) \cdot \left( \frac{PM_{i,t}}{PR_{i,t}} \right) \right]^{\frac{1}{1 - \rho^{\sigma}}} \quad (C.53)
\]

\[
QHIR_{I,t} = \gamma_{i}^{fi} \left[ \delta^{hi}_{i} QHR_{i,t}^{\rho_{i}} + \delta^{hi}_{i}QHI_{I,t}^{\rho_{i}} \right]^{rac{1}{\rho_{i}}} \quad (C.54)
\]

\[
\frac{QHR_{i,t}}{QHI_{I,t}} = \left[ \left( \frac{\delta^{hi}_{i}}{1 - \delta^{hi}_{i}} \right) \cdot \left( \frac{PI_{i,t}}{PR_{i,t}} \right) \right]^{\frac{1}{1 - \rho^{\sigma}}} \quad (C.55)
\]

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Government

\[ FD_t = G_{Pt} + \sum_{d\in gins} TRG_{d\in gins,t} \cdot PC_t - \]

\[ \left( d^p \cdot \sum_i rki,t \cdot K_{i,t} + \sum_i IBT_i,t + \sum_i L_{j,t} \cdot w_t + FE_{\epsilon_t} \right) \]  \hspace{1cm} (C.56)

\[ QG_{i,t} = \delta^q_i \cdot G_t \]  \hspace{1cm} (C.57)

\[ QGR_i,t = QG_i,t; QGM_i,t = 0; \]  \hspace{1cm} (C.58)

Investment demand

\[ QV_{i,t} = \sum_j KM_{i,j} \cdot J_{j,t} \]  \hspace{1cm} (C.59)

\[ QV_{I,t} = \gamma^v_i \left[ \delta^{qvm} QVM_{i,t}^{\gamma^A} + (1 - \delta^{qvr}) QVR_{i,t}^{\gamma^A} \right]^{\frac{1}{\gamma^A}} \]  \hspace{1cm} (C.60)

\[ \frac{QVM_{i,t}}{QVR_{i,t}} = \left[ \left( \frac{\delta^{qvm}}{\delta^{qvr}} \right) \cdot \left( \frac{PIR_{i,t}}{PM_{i,t}} \right) \right]^{\frac{1}{1-\rho^A}} \]  \hspace{1cm} (C.61)

\[ QVR_{I,t} = \gamma^{vir}_i \left[ \delta^{qvi} QVI_{i,t}^{\gamma^A} + (1 - \delta^{qvr}) QVR_{i,t}^{\gamma^A} \right]^{\frac{1}{\gamma^A}} \]  \hspace{1cm} (C.62)

\[ \frac{QVR_{I,t}}{QVI_{i,t}} = \left[ \left( \frac{\delta^{qvr}}{\delta^{qvi}} \right) \cdot \left( \frac{PI_{i,t}}{PR_{i,t}} \right) \right]^{\frac{1}{1-\rho^A}} \]  \hspace{1cm} (C.63)

Time path of investment

\[ J_{i,t} = I_{i,t} \left( 1 - bb - tk + \frac{\beta}{2} \left( \frac{I_{i,t}}{K_{i,t}} - \alpha \right)^2 \right) \]  \hspace{1cm} (C.64)

\[ \frac{I_t}{K_t} = \alpha + \frac{1}{\beta} \cdot \left[ \frac{\delta_{i,t}}{PK_t} - (1 - bb - tk) \right] \]  \hspace{1cm} (C.65)
\[ \dot{\lambda}_{i,t} = \lambda_{i,t}(r_t + \delta) - R_{i,t}^k \]  
(C.66)

\[ \theta(x_t) = \frac{\beta (x_t - \alpha)^2}{2 x_t}; \text{and } x_t = \frac{x_t}{k_t} \]  
(C.67)

\[ R_{i,t}^k = r k_t - P k + t \left[ \frac{I_{i,t}}{K_{i,t}} \right]^2 \theta'_t(I/K) \]  
(C.68)

Factors accumulation

\[ KS_{i,t+1} = (1 - \delta)KS_{i,t} + I_{i,t} \]  
(C.69)

\[ K_{i,t} = KS_{i,t} \]  
(C.70)

\[ LS_t \cdot (1 - u_t) = \sum_j L_{j,t} \]  
(C.71)

Indirect taxes and subsidies

\[ IBT_{i,t} = btax_i \cdot X_{i,t} \cdot PQ_{i,t} \]  
(C.72)

Total demand for import and current account

\[ M_{i,t} = \sum_i VI_{i,j,t} + \sum_i VM_{i,j,t} + \sum_i QHM_{i,h,t} + QGM_{i,t} + QVI_{i,t} + QVM_{i,t} \]  
(C.73)

\[ TB_t = \sum_i M_{i,t} \cdot PM_{i,t} - \sum_i E_{i,t} \cdot PE_{i,t} + \epsilon \left( \sum_{dugsins} REM_{dugind} + FE \right) \]  
(C.74)

Assets
\[ VF_{i,t} = \lambda_{i,t} \cdot K_{i,t} \]  \hspace{1cm} (C.75)

\[ D_{t+1} = (1 + r) \cdot D_t + TB + t \]  \hspace{1cm} (C.76)

\[ Pg_{t+1} \cdot GD_{t+1} = \left[ 1 + r + \left( \frac{Pc_{t+1}}{Pc_t} - 1 \right) \right] \cdot PG_t \cdot Gd_t + FD_t \]  \hspace{1cm} (C.77)

**Steady state conditions**

\[ \delta \cdot KS_{i,T} = I_{i,t} \]  \hspace{1cm} (C.78)

\[ R^k_{i,T} = \lambda_{i,T}(r + \delta) \]  \hspace{1cm} (C.79)

\[ FD_t = \left[ 1 + r + \left( \frac{Pc_{t+1}}{Pc_t} - 1 \right) \right] \cdot PG_t \cdot Gd_t \]  \hspace{1cm} (C.80)

\[ TB_T = r \cdot D_t \]  \hspace{1cm} (C.81)

\[ NFW_t \cdot r = (1 - \tau_t)L^s_t(1 - u_t)w_t + Trf_t \]  \hspace{1cm} (C.82)

\[ FW_t \cdot r = \Pi - S_t + Trf_t \]  \hspace{1cm} (C.83)

**To produce short-run and long-run results**

\[ KS_{i,t=1} = KS_{i,t=0} \]  \hspace{1cm} (C.84)

\[ LS_{i,t=1} = LS_{i,t=0} \]  \hspace{1cm} (C.85)
\[ GD_{i,t=1} = GD_{i,t=0} \quad \text{(C.86)} \]

\[ D_{i,t=1} = D_{i,t=0} \quad \text{(C.87)} \]

**Glossary**

**Set**

\( i, j \quad i = j \) the set of goods or industries

\( \text{ins} \) the set of institutions

\( \text{dins}(\subset \text{ins}) \) the set of domestic institutions

\( \text{dngins}(\subset \text{dins}) \) the set of non-government institutions

\( \text{fins}(\subset \text{dins}) \) the set of foreign institutions

\( E(\subset i) \) the set of energy sectors Electricity, Gas, Oil and Coal

\( NE(\subset i) \) the set of non-energy

**Prices**

\( PY_{i,t} \) value added price

\( PR_{i,t} \) regional price

\( PQ_{i,t} \) output price

\( PIR_{i,t} \) national commodity price(regional+RUK)

\( w_t \) unified nominal wage

\( wb_t \) after tax wage

\( rk_{i,t} \) rate of return to capital

\( Pk_t \) capital good price

\( UCK_t \) user cost of capital

\( \lambda_t \) shadow price of capital

\( Pc_t \) aggregate consumption price

\( Pk_t \) aggregate price of Government consumption goods

\( Pk_t \) exchange rate (fixed)

**Endogenous variables**
\(X_{i,t}\) total output
\(R_{i,t}\) regional supply
\(M_{i,t}\) total import
\(E_{i,t}\) total export (interregional+regional)
\(Y_{i,t}\) value added
\(L_{i,t}\) labour demand
\(K_{i,t}\) physical capital demand
\(KS_{i,t}\) capital stock
\(LS_{i,t}\) labour supply
\(VV_{i,j,t}\) total intermediate inputs
\(V_{i,t}\) total intermediate inputs in \(i\)
\(VR_{i,j,t}\) regional intermediate inputs
\(VM_{i,j,t}\) ROW intermediate inputs
\(VIR_{i,j,t}\) national intermediate inputs (Scotland+RUK)
\(VI_{i,j,t}\) RUK intermediate inputs
\(G_t\) aggregate Government expenditure
\(QG_{i,t}\) Government expenditure by sector \(i\)
\(QGR_{i,t}\) regional Government expenditure by sector \(i\)
\(QGM_{i,t}\) national Government expenditure by sector \(i\)
\(C_t\) aggregate household consumption
\(Ec_t\) household consumption of energy
\(NEc_t\) household consumption of non-energy goods
\(CO_t\) household consumption of coal and oil
\(EG_t\) household consumption of electricity and gas
\(ELE_t\) household consumption of electricity
\(GAS_t\) household consumption of gas
\(CL_t\) household consumption of coal
\(OIL_t\) household consumption of oil
\(QH_{i,t}\) household consumption by sector \(i\)
\(\text{QHR}_{i,t}\) household regional consumption by sector \(i\)
\(\text{QHIR}_{i,t}\) regional+RUK consumption by sector \(i\)
\(\text{QHM}_{i,t}\) imported consumption bys sector \(i\)
\(\text{QV}_{i,t}\) total investment by sector of origin \(i\)
\(\text{QVR}_{i,t}\) regional investment by sector of origin \(i\)
\(\text{QIR}_{i,t}\) ROW investment demand by sector \(i\)
\(\text{QVI}_{i,t}\) RUK investment demand by sector \(i\)
\(I_{j,t}\) investment by sector of destination \(j\)
\(J_{j,t}\) investment by destination \(j\) with adjustment cost
\(u_t\) regional unemployment rate
\(u^N_t\) national unemployment rate
\(R^k_{i,t}\) marginal revenue of capital
\(S_t\) domestic non-government savings
\(\text{Trf}_{t}\) household net transfer
\(\text{Trsf}_{dngins,dnginsp,t}\) transfer among \(dngins\)
\(\text{HTAX}_t\) total household tax
\(\text{TB}_t\) current account balance

**Exogenous variables**
\(\text{REM}_t\) remittance for \(dngins\)
\(\text{FE}_t\) remittance for Government
\(\text{GSAV}_t\) Government savings
\(r\) interest rate

**Elasticities**
\(\sigma\) constant elasticity of marginal utility
\(\rho^X_i\) elasticity of substitution between intermediate and value added
\(\rho^Y_i\) elasticity of substitution between capital and labour
\(\rho^A_i\) elasticity of substitution in Armington function
\(\sigma^e_i\) elasticity of export with respect to term trade
\(\sigma^e_i\) substitution in consumption between energy and non-energy
$\sigma_{i}^{g}$ substitution in consumption between CO and EG

$\sigma_{i}^{o}$ substitution in consumption between coal and oil

$\sigma_{i}^{el}$ substitution in consumption between electricity and gas

**Parameters**

$\alpha_{i,j}^{V}$ input-output coefficients for $i$ used in $j$

$\alpha_{j}^{V}$ share of value added in production

$\delta_{j}^{Y,V}$ share in CES output function in sector $j$

$\delta_{i,j}^{k,l}$ share in value added function in sector $j$

$\delta_{i,j}^{vir,vm,vi}$ share in CES function for intermediate goods

$\delta_{i,j}^{qvir,qvm,qvi}$ share in CES function for investment

$\delta_{i,j}^{E,co,cl}$ share in CES function for household consumption

$\delta_{i,j}^{hr,hm}$ share in CES function for household consumption

$\delta_{i,j}^{gr,gm}$ share in CES function for Government consumption

$\gamma_{i,j}^{vir}$ shift parameter in CES for intermediate goods

$\gamma_{i,j}^{f}$ shift parameter in CES for household consumption

$\gamma_{i,j}^{g}$ shift parameter in CES for Government consumption

$btax_{i}$ rate of business tax

$KM_{i,j}$ physical capital matrix

$mps$ rate of saving $dngins$

$\tau$ rate of income tax

$\rho$ pure rate of consumer time preference

$bb$ rate of distortion or incentive to invest

$\delta$ depreciation rate