More dangerous than dyads: bargaining and war in multi-actor disputes

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MORE DANGEROUS THAN DYADS: BARGAINING AND WAR IN MULTI-ACTOR DISPUTES

ABSTRACT. For the bargaining model of war, in the absence of incomplete information and commitment problems, war is irrational. But this finding rests on a simple and rarely discussed assumption – that bargaining is between exactly two participants. When we relax this assumption, in a three-player bargaining game, war is an equilibrium. Thus, a key finding of the bargaining model – that there is always an agreement that all states prefer to war – is an artifact of dyadic analysis. By removing this limitation, we can find new factors that affect the risk of war: the number of actors, divergence in state preferences, alliance dynamics, and the issue being bargained over.

Date: June 10, 2016.
In recent years, a number of international conflicts have been particularly intractable because they involve many parties. It has been difficult for the United States to achieve its aims in Syria’s civil war partially because of Syria’s close ties to Russia. The struggle over Iran’s nuclear program has been particularly fraught for the Obama administration, trying to avoid an Israeli attack on Iran on the one hand, and Iranian acquisition of nuclear weapons on the other. Militarized Interstate Disputes involving more than two actors are about 7 times as likely (0.15 probability of war) to escalate to war as purely dyadic cases (0.02 probability) (Ghosn, Palmer and Bremer, 2004).\(^1\) Yet despite the proliferation of cases in which multiple actors exert an effect on the outcome, our frame of analysis in international relations is predominantly dyadic.\(^2\) There has been some discussion of the problematic statistical property of dyads\(^3\), but there has been less discussion of the fact that our theories of interstate conflict are also dyadic.

The current dominant explanations for conflict are generated from the bargaining model of war. This model has been used to motivate empirical work on the onset, duration and recurrence of interstate and civil war; to show how factors such as economic interdependence and foreign support for insurgency lead to war; and even to explain non-conflictual phenomena such as sanctions and the international criminal court (Walter, 1999, 2009; Werner and Yuen, 2005; Fortna, 2003; Schultz, 2010; Polachek and Xiang, 2010; Gartzke and Boehmer, 2001; Drezner, 2003; Simmons and Danner, 2010). A major insight of the bargaining model is that war is \textit{ex post} irrational for both states; in this model if there is conflict, either incomplete information has obscured a mutually beneficial settlement, or states are unable to credibly commit to abide by it.\(^4\)

I find that this key finding of the bargaining model – that there is always that agreement all states prefer to war – an artifact of limiting the analysis to two players.\(^5\) When there are more

\(^1\)If we examine only cases with 3 actors, the probability that a case ends in war is about .1.
\(^2\)See Bremer (1992) or Bennett and Allan Stam, III (2004) for prominent examples.
\(^3\)See Erikson and Rader (2014), Poast (2010), and Dorff and Ward (2012) for a discussion of the danger of using dyadic analysis to understand a non-dyadic phenomenon.
\(^4\)Issues indivisibility is sometimes posited as a third explanation for war, but as Powell (2006) shows, this is a special case of a commitment problem.
\(^5\)This finding is analogous to other theoretical findings about the difference between two-player dynamics and those involving more players. The median voter theory’s equilibrium breaks down when we introduce a third
than two players, the players’ preferences regarding the issue will sometimes be so disparate that there is no way to reach an agreement that will dissuade all of them from fighting. In the rest of this paper I briefly discuss the bargaining model of war, I model a simple three player bargaining game, discuss the main findings of this game, examine the equilibria of the game to show how war is possible, and discuss comparative statics generated by simulation.

1.1. Current Bargaining Models. The use of the bargaining model in the study of conflict has generally focused on two major explanations for war: incomplete information and commitment problems. These explanations for war have generally held whether bargaining is conceptualized as a state making a take it or leave it offer, as in Fearon (1995), or as a set of alternating offers, as in Powell (1999).

When there is a commitment problems, a bargain that may be beneficial to all players presently will become unacceptable to one player in the future. The shadow of the future renders conflict preferable to such an agreement. (Powell, 2006). Informational asymmetry occurs because states may be poorly informed about their opponent’s capabilities or resolve (Fearon, 1995). As stronger, more resolved states receive a better outcome in a settlement, there is incentive for misrepresentation. States face a risk/reward trade-off: if the state is sufficiently accommodating, the dispute will be resolved peacefully, but occasionally, the state can do better by making a smaller offer, which will lead to war if they face an unexpectedly resolute foe.

There have been some models which expand bargaining beyond the dyad. Most of this work operates under the framework of incomplete information – they examine how additional players modify informational requirements for peace or endogenously change the games informational content. Many of these models focus on domestic constituencies affecting bargaining leverage, and the incorporation of a domestic actor generally has been found to help a state reveal their resolve, get a larger slice of the pie, and avoid war (Putnam, 1988; Fearon, 1994; candidate (Patty, Snyder and Ting, 2009). In parliamentary bargaining over coalition formation, circumstances with only two major parties are uninteresting and easy to solve, whereas the measures that parties use to choose partners in multiparty settings are the subject of serious academic debate (Laver, de Marchi and Mutlu, 2011).
Schultz, 1998). Similarly, third party mediators are seen as ameliorating incomplete information (Kydd, 2006).

This paper steps back and examines the effect of an additional player on the simplest and most peace-prone model: a model with complete information and an absence of commitment problems. This paper’s findings stand in contrast with those of most of these models. Indeed, most models focus on third parties ameliorating informational issues and making war less likely, whereas the model presented here demonstrates how third players can make war unavoidable even with complete information.

2. The Model

2.1. Players. In this three-player bargaining model, there is a policy dispute that can be settled through either peaceful negotiation or war. Each player in the model is a state with both an interest in the disputed issue and the ability to fight to see its preferred outcome enacted. In this model, each of these states has three relevant characteristics: a preference over the disputed issue, military capabilities, and the cost associated with going to war.

In this model, in contrast to many two-player bargaining models, states are bargaining over the preferred outcome of a policy issue rather than purely distributional concerns. In two-player bargaining, the decision does not matter; in fact the two choices are isomorphic. In Reiter (2003)’s review of bargaining models of war, he treats the two in tandem, discussing “disagreement over resource allocation and/or policy choice.” When we add any number of additional players, the concepts diverge. Each state has a preferred policy outcome, or ideal

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6With the exception of Cunningham (2006)’s examination of the relationship between the number of rebel groups and civil war termination. Where this paper differs from Cunningham’s work is that the model in this paper concerns war onset, applies to contexts beyond intrastate war, and provides an explicit means of determining how different factors affect the likelihood of war as the number of players changes.

7By making this decision, I am unable to represent disputes in which each player is simply trying to maximize its share of an excludable good, such as the division of Poland among Prussia, Austria, and Russia. If one believes that most disputes, or important disputes, are purely distributional, this model will not be useful. I would argue that many disputes and conflicts could be better represented as disputes over policy. Moreover, bargaining over policy allows me to represent situations in which players have similar preferences and in which the benefits to players of policy concessions are non-excludable. The model can thus represent policies, such as Iran’s nuclear program and Pakistan’s decision to aid the United States in the war on terror, as well as disputes over the division of a unit of a good between two players, such as the division of Crimea or the Diaoyu/Senkaku islands.
point, which I represent as a point in a one-dimensional issue space.\textsuperscript{8} Without loss of generality, we order the states such that the ideal point for state 1, $s_1$, is 0, $s_2$ is larger than $s_1$, and $s_3$ is larger still. At the end of the game, each state will lose utility based on how far the final outcome is from its ideal point.

If states choose to fight rather than to successfully negotiate, the outcome of war is a costly lottery, with the odds determined by the states’ capabilities.\textsuperscript{9} Each state $i$ has a capability $k_i$ where $k_i > 0$. If state $i$ were to fight a war against state $j$, the probability that state $i$ would win is $\frac{k_i}{k_i + k_j}$, and if all three players were involved, the probability that state $i$ would triumph would be $\frac{k_i}{k_i + k_2 + k_3}$.\textsuperscript{10} If a state wins in a war, that state determines the policy and sets it to its ideal point. Each state that is involved in war will also pay a cost $c_i$ for player $i$ such that $c_i > 0$. This cost represents the deadweight loss of war, in men and materiel, and it can also be interpreted as a measure of a state’s resolve – if a state has high costs of war, then the policy benefits will seem less impressive compared with the costs, and the state will not be resolute on the issue, whereas a low cost implies that a state cares deeply about the issue.

Each player’s utility from the game is $1 - E(\text{Policy Distance}) - \text{Any Cost of War}$.

\subsection*{2.2. Sequence of Play.}
The sequence of play is represented by the game tree in figure 1. The game has three main stages: the generation of a policy offer, acceptance or rejection of that offer, and then the possible outcomes of conflict. Note that if the setup of the model were exactly the same except that bargaining concerned distributional issues exclusively, the results would be markedly different and less interesting. In particular, as shown in the appendix, if three players bargain over a unit of goods, then in a game with complete information and an absence of commitment issues, there will always be a way of dividing the good such that all players will prefer to go to war. Ironically, this is the opposite distinction drawn in Ray (2009), who found that excludable goods are more prone to conflict than are public goods.\textsuperscript{8}

\textsuperscript{9}On the one hand, as Wagner (2000) has noted, the model loses verisimilitude by abstracting war into a lottery. On the other hand, the discussions of war as a continuous bargaining process have focused on how war fighting eventually leads to informational convergence, and that with complete information, models with war as a lottery or a bargaining process have yield the same results – successful initial bargaining (Smith and Stam, 2004; Slantchev, 2003). An examination of the effect of incomplete information in three-player war would have to seriously consider a more complex view of war, but since because this paper focuses on the complete information case, I believe this simplification is warranted.

\textsuperscript{10}By using capabilities to determine probabilities of victory, rather than simply having $p_{12} = P(\text{State 1 defeats state 2})$ and so on to determine probabilities of victory, I restrict the possible outcomes of conflict and attempt to guard against the equilibria being the resultings of from corner cases where probabilities of victory are intransitive. This approach also helps build a closer link between the parameters in the model to observable factors for empirical analysis.
Figure 1. Sequential form for three-player bargaining. The information sets represent the simultaneous decision to accept an offer or attack another player. The terminal nodes are labeled with the number of players involved in the final conflict or \( \emptyset \) if the final outcome is peaceful.

offer, and the decision by any nonbelligerent players to join a conflict or abstain from joining the conflict.

A key distinction between the sequence of play in this model, and in most two-player bargaining models, is that I do not assign one of the states the responsibility of choosing the negotiated settlement offer. Rather, I exogenously generate an offer by using a decision rule chosen to maximize the likelihood of peace. Such a rule is used to prevent the results of the model from being driven by the choice in bargaining protocol.\(^{11}\) In particular, the initial bargaining offer follows two rules:

\(^{11}\)Scholars have examined the sensitivity of the bargaining model’s results (in particular, results concerning incomplete information) to complicating the bargaining process. Leventoglu and Tarar (2008) demonstrated that giving the less advantaged state the opportunity to make a counteroffer and fight if it was rejected allowed the bargaining back and forth to better inform players and avoid war. In contrast, Langlois and Langlois (2006) demonstrated that in the presence of alternating offers and the ability to continue bargaining while fighting, conflict could arise even while states were fully informed because a revisionist state must probabilistically threaten war in to spur concessions. This policy offer is the results of from some bargaining process, though but I remain agnostic as to regarding whether it is “take-it-or-leave-it” bargaining as in Fearon (1995), or some form of alternating offers as in Powell (1999) or Leventoglu and Tarar (2008).
(1) If there is an offer that satisfies all three players and that leads to a peaceful outcome, choose the median of these successful offers.

(2) If no offer exists that satisfies all three players, offer a policy equivalent to the median player’s ideal point ($s_2$).

We could think of this as the offer being made by an unbiased mediator whose only interest is to avoid war. Alternatively, one could think of this as being analogous to the role of the auctioneer in a Walrasian market equilibrium, a player whose only interest is to maximize the efficiency of the outcome.

Once the initial offer is made, each state simultaneously chooses whether to accept the offer and remain peaceful or to reject the offer by attacking one of the other players. If each player chooses to accept the offer, the game has a peaceful and efficient outcome and each player receives utility based on the distance between the initial offer $x$ and the respective ideal point. Thus, for state $i$, the utility from accepting $x$ is:

\begin{equation}
U_i(x) = 1 - |x - s_i|
\end{equation}

If the offer is rejected by at least one player, the outcome of the game will be war. In particular, the game will end in a war involving all states that either attacked another player or were attacked. If all three states are participants in the war (for example, because states 1 and 2 both attacked state 3 or because state 1 attacked state 2, which attacked state 3), then state 1 (for example) will receive the following utility from a conflict involving all players:

\begin{equation}
U_1(W_{123}) = 1 - \frac{k_2 s_2}{k_1 + k_2 + k_3} - \frac{k_3 s_3}{k_1 + k_2 + k_3} - c_1
\end{equation}

or unity less the distance between state one’s ideal point and the other states’, weighted by the probability that each other state will triumph, less the cost of war to state one.
If all three states are involved in war, the game terminates; similarly, the game ends if all players accept the initial offer. However, if only two players are involved in conflict, the third player can choose to either join the conflict or stay out of it. I allow for this additional decision to allow for potential dynamics of deterrence in the model. For example, if the United States is willing to accept an offer concerning the Taiwan Strait rather than fight but if China would prefer a dyadic war with Taiwan, I would like the model to be able to capture the ability of the United States to deter China with the threat of a three-party war. Given that joining a conflict is an additional decision, states can include calculations about the actions of a third player when deciding whether they prefer war to the proffered settlement. If the third state joins, the outcome is as discussed above, that is, a general three-player war, whereas if this state stays aloof and refuses to join, the outcome will be a dyadic war. The utility for a participant in the dyadic war (between state \( i \) and \( j \)) is as follows:

\[
U_i(W_{ij}) = 1 - \frac{k_j|s_i - s_j|}{k_i + k_j} - c_i
\]

While the excluded player does not fight, it is still affected by the outcome because of its concern in the disputed policy. The utility for the uninvolved state (player \( l \)) is

\[
U_l(W_{ij}) = 1 - \frac{k_i|s_i - s_l|}{k_i + k_j} - \frac{k_j|s_j - s_l|}{k_i + k_j}
\]

Three assumptions merit discussion for the sequence of play in this game: the informational environment, the commitment technology used, and the possibility of alliance dynamics if all three players are involved in conflict.

This game is one of complete information – players are fully informed of every other player’s capability, preference, and cost of conflict. This assumption is not made because of any belief that informational issues do not matter. In fact, one could argue, as Cunningham (2006) and Esteban and Ray (2001) do, that the severity of informational issues increases as
the number of players increases. Rather, I assume perfect information to demonstrate that even when the normal mechanisms for war are gone, we can still have conflict in three-player bargaining. Similarly, I attempt to minimize the role of commitment problems. In this game, there is no shadow of the future affecting the players’ decisions, and they are blessed with perfect commitment technology: if the players are able to agree to a compromise, this compromise will take place with perfect enforcement. Now, it is true that when viewed through a dyadic lens, there may be mutually agreeable settlements that two states cannot commit to, but this is less because either state has an incentive to renege on the agreement and more because the third party may attack and spoil the agreement.

If all three states are involved in a war, the war will be modeled as one of “all against all.” Each state will have an independent probability of triumph and associated utility. One could argue that the model ignores coalition dynamics: two states with similar preferences might decide to work together to ensure a mutually beneficial outcome. This is less of an issue precisely because of the decision to model bargaining as the determination of a policy on a policy space. If two players have very similar preferences, they effectively form an endogenous alliance: if either of them were to win, they would set the policy to their ideal point, and both members of this pseudo-alliance thus would be happy. However, alliances may actually be more effective than they appear just from looking at their combined capabilities. Thus, I also examine an extension to the model in which states formally choose sides and members of the two-state alliance have an additional likelihood of defeating their lone opponent. This model is somewhat more complicated, but I can show that unless the added benefit of fighting in an alliance is particularly large, the allianceless model’s main results will hold.

3. SOME FINDINGS FROM THREE-PLAYER BARGAINING

In a two-player bargaining game with perfect information and an absence of commitment problems, the core result is quite simple – both players will always accept a peaceful settlement. The results of three-player bargaining are not so simple. In particular, there are three major
results that warrant discussion: the existence of cases where no possible settlement is preferable to dyadic conflict, the existence of cases where dyadic war is the outcome and the third actor chooses not to intervene, and the existence of equilibria where all players fight.

3.1. **Offers, Dissatisfaction, and Dyadic War.** In this three-player bargaining game, states are confronted with an initial choice: accept a compromise, or fight a dyadic war. In canonical two-player bargaining games, offers that causes all players to reject violence and settle peacefully will always exist. This remains true in this model if you simply remove one of the three players. In the two-player variant of this model, state 1 will accept an offer if it preferable to her outside option and thus

\[
1 - |x| > 1 - \frac{s_2 k_2}{k_1 + k_2} - c_1
\]

implying that \( x \in (-\frac{s_2 k_2}{k_1 + k_2} - c_1, \frac{s_2 k_2}{k_1 + k_2} + c_1) \). Similarly, state 2 will accept an offer if \( x \in (\frac{s_2 k_2}{k_1 + k_2} - c_2, s_2 + \frac{s_2 k_2}{k_1 + k_2} + c_2) \). We thus have a guaranteed range of agreement \((\frac{s_2 k_2}{k_1 + k_2} - c_2, \frac{s_2 k_1}{k_1 + k_2} + c_1)\) because the cost of war is always positive. Therefore, in the two-player game, there always exists an offer that is mutually preferable to dyadic war. This ceases to be the case when a third player is introduced.

When we introduce a third player, each player gains an additional target for attack, and thus, an additional outside option. The introduction of a third player renders it more difficult for an offer to be preferable to dyadic war. State 1 will prefer to accept an offer \( x \) rather than fighting states 2 and 3 if and only if both

\[
1 - |x| \geq 1 - \frac{s_2 k_2}{k_1 + k_2} - c_1
\]

\(^{12}\)Technically, there are is a range of values that will also satisfy player 1 which that are smaller than 0, and equivalent values that satisfy player 3 which that are greater than \( s_3 \), but for simplicity’s sake, I limit my analysis to offers that are in the range of the three player’s ideal points.
and

\[(7) \quad 1 - |x| \geq 1 - \frac{s_3 k_3}{k_1 + k_3} - c_3 \]

Therefore, player 1 will accept \(x\) only if:

\[(8) \quad 0 \leq x \leq \min\left(\frac{s_2 k_2}{k_1 + k_2}, \frac{s_3 k_3}{k_1 + k_3}\right) + c_1 \]

We denote the set of values of \(x\) that can satisfy equation 8 as \(X_1\). Similarly, player 2 will only accept an offer \(x\) if:

\[(9) \quad x \in \left(s_2 - \min\left(\frac{s_2 k_1}{k_1 + k_2}, \frac{(s_3 - s_2) k_3}{k_2 + k_3}\right) - c_2, s_2 + \min\left(\frac{s_2 k_1}{k_1 + k_2}, \frac{(s_3 - s_2) k_3}{k_2 + k_3}\right) + c_2\right) \]

The range of values for \(x\) in equation 9 is denoted \(X_2\). Player 3 will accept an offer \(x\) if and only if

\[(10) \quad s_3 - \min\left(\frac{s_3 k_1}{k_1 + k_3}, \frac{(s_3 - s_2) k_2}{k_2 + k_3}\right) - c_3 \leq x \leq s_3 \]

which is henceforth referred to as \(X_3\).

For notational purposes, we denote the set \(\{s_2, s_3, k_1, k_2, k_3, c_1, c_2, c_3\} \equiv \Theta\). The relationship between \(\Theta\) and equations 8, 9, 10 is the subject of the first proposition.

**Proposition 1.1:** \(\exists \Theta\), such that \(\nexists x \in X_1 \cap X_2 \cap X_3\). Thus, for certain values of \(\Theta\), \(x\) cannot satisfy equations 8, 9, and 10, because \(X_1 \cap X_2 \cap X_3 = \emptyset\).

It is not possible to satisfy the minimal demands of all three players for some parameter configurations. In these cases, the equilibrium outcome will be war. This proposition does not necessarily imply that war will be the outcome of the game; the utility of war will be conditioned on whether the excluded player (the player who is not attacked by the dissatisfied paper) intervenes. What the proposition does show, however, is that there will not always be a negotiated settlement that is jointly preferable to particular dyadic wars.
We prove existence here by demonstrating one case where the three constraints cannot simultaneously hold. For a more complete list of cases, see section 1 in the technical appendix.

Consider a conflict between equation 8 and equation 9 and in particular, consider the cases where player 1 is considering attacking player 2 and player 3 is considering attacking player 1. In other words that
\[ X'_{1} = [0, \frac{s_{2}k_{2}}{k_{1}+k_{2}} + c_{1}], \]
\[ X'_{3} = [s_{3} - \frac{s_{2}k_{2}}{k_{1}+k_{3}} - c_{3}, s_{3}]. \]
By the definitions of \( X_{1} \) and \( X_{3} \), they will be subsets of \( X'_{1} \) and \( X'_{3} \).

For certain values of \( \Theta, \not\exists x \) such that \( x \in X'_{1} \) and \( x \in X'_{3} \). This is equivalent to saying that the larger edge of \( X'_{1} \) is smaller than the smaller edge of \( X'_{2} \), or

\[ c_{1} + c_{3} < \frac{s_{3}k_{3}}{k_{1}+k_{3}} - \frac{s_{2}k_{2}}{k_{1}+k_{2}} \]

This will hold, implying existence for Proposition 1.1 if \( s_{3} \) and \( k_{3} \) are sufficiently large. Because neither value is constrained, we have demonstrated existence of a set \( \Theta \) wherein the parameters make it impossible to generate a compromise \( x \) that is jointly preferable for player 1 and player 3 to starting dyadic wars with player 2 and player 1, respectively. Thus Proposition 1.1 is proved.

Of course, this does not mean that compromise will always be inferior to dyadic conflict. For many values of \( \Theta \), bargaining will occur just as it did in the canonical two-player game, with all players accepting the initial offer. Proposition 1.2 shows the existence of these cases.

**Proposition 1.2:** \( \exists \Theta, \text{ such that } \exists x \in X_{1} \cap X_{2} \cap X_{3}. \) Thus, for certain values of \( \Theta, x \) can satisfy equations 8, 9, and 10 because \( X_{1} \cap X_{2} \cap X_{3} \neq \emptyset. \)

Proposition 1.2 is the complement of Proposition 1.1, showing that for some constellations of parameters we can satisfy the minimal demands of each player and avoid war. Proving Proposition 1.2 requires that I find a set of values such that constraint 11 does not hold, and this only involves sufficiently large values of \( c_{1} \) or \( c_{3} \). Thus existence is simple to demonstrate.

\[ \text{The sets will be identical if } \frac{k_{3}s_{3}}{k_{1}+k_{2}} > \frac{k_{2}k_{2}}{k_{1}+k_{3}} \text{ and } \frac{(s_{3} - s_{2})b_{2}}{k_{1}+k_{2}} > \frac{s_{2}k_{2}}{k_{1}+k_{3}}, \text{ if either inequality does not hold, } X_{1} \text{ and } X_{3} \text{ will be smaller subsets, and obviously, if two sets do not intersect, the subsets will not intersect.} \]
3.2. Entering a Dyadic War. In those cases where we there is a set of parameters such that no offer is jointly preferable to dyadic war, we then want to examine whether such a war is a possible equilibrium outcome. We would like to know whether the excluded player will enter a dyadic war (making it a general war) or abstain, causing it to remain dyadic.

If player 1 and player 2 were fighting, player 3 would enter the conflict if and only if

\[ 1 - \frac{k_1 s_3}{k_1 + k_2} - \frac{k_2 (s_3 - s_2)}{k_1 + k_2} < 1 - \frac{k_1 s_3}{k_1 + k_2 + k_3} - \frac{k_2 (s_3 - s_2)}{k_1 + k_2 + k_3} - c_3 \]

which can be rewritten as:

\[ c_3 < s_3 \left( \frac{k_1}{k_1 + k_2} - \frac{k_1}{k_1 + k_2 + k_3} \right) + (s_3 - s_2) \left( \frac{k_2}{k_1 + k_2} - \frac{k_2}{k_1 + k_2 + k_3} \right) \]

To understand the dynamics of bargaining, we are primarily interested in the players’ choices to enter the conflict when rejecting an offer is rational and when they are on the equilibrium path, that is, when \( \Theta \) is such that Proposition 1.1’s existence result holds or at least one player prefers a dyadic war to any possible offer. When this existence result holds, for every offer \( x \), there will be one player who chooses to attack, one player who is the target, and one player who can either choose to enter the conflict or abstain from fighting. We term the attacker, the defender, and the potential joiner \( P_A(x), P_T(x), P_J(x) \), respectively.

**Proposition 2:** \( \exists \Theta, such that \exists x \in X_1 \cap X_2 \cap X_3 \) and such that \( P_J(x) \) plays “Not Join” for all \( x \in (0, s_3) \).

This proposition shows that in some cases, the equilibrium of the game will be such that two players fight, and the third player does not. To demonstrate existence, we again turn to the case where \( \Theta \) is such that \( X'_1 \cap X'_3 \). In this case, if \( x \in (0, \frac{k_1 s_2}{k_1 + k_2} + c_1) \), then \( P_1 = P_A(x), P_2 = P_T(x), P_3 = P_J(x) \) and if \( x \in \left( \frac{s_3 k_2}{k_1 + k_3} - c_3, s_3 \right) \), then \( P_3 = P_A(x), P_1 = P_T(x), P_2 = P_J(x) \). If neither holds, then we can have subgames in which either player 3 attacks player 1 or player 1 attacks player 2. We thus need to investigate the decision of whether to join for players 2 and 3. For player 2, the inequality that determines this decision is
\[(14) \quad c_2 < s_2\left(\frac{k_1}{k_1 + k_3} - \frac{k_1}{k_1 + k_2 + k_3}\right) + (s_3 - s_2)\left(\frac{k_3}{k_1 + k_3} - \frac{k_3}{k_1 + k_2 + k_3}\right)\]

and for player 3, the inequality is described in equation 13. We are therefore looking for a set \(\Theta\) that simultaneously satisfies inequality 11 and fails to satisfy inequalities 13 and 14. Failing to satisfy inequality 14 is simple: \(c_2\) does not appear in the other two inequalities; therefore, we are free to assign it any value greater than 0, and for a sufficiently high value of \(c_2\), we ensure that player 2 will not join a conflict. For the calculation of whether player 3 joins, we can demonstrate existence by taking the lowest possible cost of war for which player 3 plays “Not Join” and determining whether inequality 11 is still possible to satisfy. With some algebra, we can substitute and rearrange the inequality to provide a new constraint. There will be no value \(x\) that satisfies inequality 11 and player 3 will not intervene if

\[(15) \quad c_1 < \frac{s_2 k_2}{k_1 + k_2} + \frac{s_3 (k_1 + k_2)}{k_1 + k_2 + k_3} - \frac{s_3 k_3}{k_1 + k_3} - \frac{s_2 k_1}{k_1 + k_2} - \frac{s_2 k_2}{k_1 + k_2 + k_3}\]

To show that values of \(\Theta\) exist such that this inequality holds, note that as \(k_2\) increases, the limit tends to

\[(16) \quad c_1 < s_3 + s_2 - s_2 - \frac{s_3 k_3}{k_1 + k_3} = s_3 k_1 \frac{k_1}{k_1 + k_3}\]

Thus, existence is proved for Proposition 2. Further, we have proved the existence of equilibria for \(\Theta\) that satisfy the preceding constraints as follows. Nature proposes a compromise \(x^* \in (0, s_3)\). If \(x^* > \frac{k_2 s_2}{k_1 + k_2} + c_1\), player 3 attacks player 1, and player 2 abstains from entering the conflict. If \(x^* < \frac{k_2 s_2}{k_1 + k_2} + c_1\), player 1 attacks player 2 and player 3 abstains from entering the conflict. Here, we see one of the major results of three-player bargaining and one of the crucial differences from traditional dyadic models: despite the inefficiency of war, complete information and perfect commitment technology, in some cases, dyadic war will be unavoidable.
3.3. **Joiners, Deterrence, and General Wars.** We finally consider the set of cases where a player would be willing to enter a dyadic conflict, and determine whether the willingness of the third player to enter a conflict will always lead to successful deterrence and peaceful equilibria or whether it will allow for equilibria in which all three players are involved in conflict.

When \( P_{j(x)} \) is willing to play “Join”, this will alter the range of values \( P_{A(x)} \) is willing to accept. For example, player 1 prefers fighting player 2 rather than accepting an offer \( x \) if

\[
x > \frac{k_2 s_2}{k_1 + k_2} + c_1
\]

but if player 3 would play join in this circumstance, player 1 will accept any offer \( x \) such that

\[
0 \leq x \leq \frac{k_2 s_2 + k_3 s_3}{k_1 + k_2 + k_3} + c_1.
\]

We call the range of values acceptable to player \( i \) if player \( j \) would play join\(^{14} \) \( X_i^{(j)} \).

**Proposition 3.1:** \( \exists \Theta \text{ such that } \not\exists X_1 \cap X_2 \cap X_3, \text{ for some } x \in (0, s_3), P_{j(x)} \text{ plays “Join,” and } \not\exists X_i \cap X_j \cap X_{i}^{(j)}. \)

This proposition posits the existence of constellations of parameters such that all three players will end up involved in conflict in equilibrium. To prove existence, we again return to our case where \( X_1' \) and \( X_2' \) have no intersection and player 2 will never intervene because of the high value of \( c_2 \). In this case, however, we ensure that player 3 will intervene because constraint 13 is satisfied. To satisfy proposition 3, no overlap between \( X_1^{(3)} \) and \( X_3' \) must exist, which implies that

\[
\frac{s_2 k_2 + s_3 k_3}{k_1 + k_2 + k_3} + c_1 < \frac{k_3 s_3}{k_1 + k_3} - c_1
\]

There exist sets of parameters \( \Theta \) that satisfies these constraints. This holds whenever:

\(^{14}\text{ Assuming that } P_j = P_{j(x)} \)
\( k_1 < \frac{k_3 s_3 - k_3 s_2}{s_2} \)

\( c_3 < \frac{k_2 k_3 s_3 - k_2 k_3 s_2 - k_1 k_2 s_2}{k_1^2 + k_1 k_2 + 2k_1 k_3 + k_2 k_3 + k_3^2} \)

\( c_1 < \frac{k_2 k_3 s_3 - c_3 k_1^2 - c_3 k_1 k_2 - 2c_3 k_1 k_3 - c_3 k_2 k_3 - c_3 k_3^2 - k_1 k_2 s_2}{k_1^2 + k_1 k_2 + 2k_1 k_3 + k_2 k_3 + k_3^2} \)

These constraints can be satisfied,\(^{15}\) and thus we have shown existence for Proposition 3. Nature offers \( x \in (0, s_3) \). If \( x \in (0, \frac{s k_1}{k_1 + k_3} - c_3) \), player 1 attacks player 2 and player 3 plays join. If \( x \in (\frac{s k_1}{k_1 + k_3} - c_3, s_3) \), player 3 attacks player 1, and player 2 plays abstain.

We complete the set of possible outcomes with Proposition 3.2, which concerns the existence of equilibria where no offer is preferable to dyadic war but the willingness of players to join the conflict ensures that the final outcome is peaceful.

**Proposition 3.2**: \( \exists \Theta \) such that, \( \forall X_1 \cap X_2 \cap X_3 \) for some \( x \in (0, s_3) \), \( P_{J(x)} \) plays “Join” and \( \exists X_i \cap X_j \cap X_{l}^{(j)} \).

This proposition will hold in the case above whenever constraint 11 is satisfied, but constraint 22 is not or when

\[
\frac{k_2 k_3 s_3 - c_3 k_1^2 - c_3 k_1 k_2 - 2c_3 k_1 k_3 - c_3 k_2 k_3 - c_3 k_3^2 - k_1 k_2 s_2}{k_1^2 + k_1 k_2 + 2k_1 k_3 + k_2 k_3 + k_3^2} < c_1 \leq \frac{s_3 k_3}{k_1 + k_3} - \frac{s_2 k_2}{k_1 + k_2} - c_3
\]

The precise algebra needed to reduce these constraints is involved, but one can ascertain that the set of values for \( c_1 \) that satisfy this constraint are not empty, which proves Proposition 3.2.\(^{16}\)

These propositions lead to four possible types of equilibria, which can all exist depending on the value of \( \Theta \). There can be peaceful equilibria because of either satisfaction with the offer

\(^{15}\)Note that the upper bound of these parameters is monotonically increasing in \( s_3 \) (which has the same effect for player 3’s decision to enter a conflict), and since the possible values are unbounded, we can always increase \( s_3 \) until the constraints are all met.

\(^{16}\)For the precise set of constraints, see the technical appendix.
given or deterrence and preference of the offer to a three-state war, there can be dyadic war, or there can be three-state war.

3.4. Extension: Formal Alliances. In the model of bargaining presented in this paper, if all three players fight, they each fight independently. I believe that this has been a defensible decision because similar preferences would in effect create endogenous alliances. However, alliances might offer benefits that make them more than the sum of their parts. Thus, we could model a conflict with all three players as a potential conflict between two alliances. In this section, I modify the final stage of the game to give an advantage to states that are fighting together and a disadvantage to the state fighting on its own. In particular, if all three players choose to fight in the first stage and if two players choose to attack the third player, they are in a de facto alliance and have an advantage in their probability of victory. Similarly, if only two players are involved in conflict in the first stage, in the final stage of the game, the excluded player may choose to do nothing (leading to a dyadic war), to enter as an independent state (leading to a war of all against all) or to enter by attacking either player (leading to a war of one state against an alliance).

The only difference in utility functions in this model is encapsulated by the term $\alpha$. Here, $\alpha$ represents the advantage of fighting together over fighting alone. It could represent increased domestic legitimacy derived from being part of a larger coalition allowing increased resource mobilization, the sharing of military strategy and technology that enables each state to punch above their weight, the difficulty of fighting a war on two fronts, or some combination of benefits of alliances. Of course, in many cases, alliances will actually be less effective the combination of the states’ capabilities might indicate. Coordination difficulties and confusion in the chain of command in regard to fighting for a common good might arise, and allies might have an incentive to hold back and let their partner do the lion’s share of the fighting and dying. For the sake of this extension, I assume that there is some benefit to being in an alliance, and thus $\alpha$ is positive. The way $\alpha$ affects the outcome of conflict is that if states $i$ and $j$ are allied against state $l$, the probability that $i$ will triumph will be $\frac{k_i + \alpha}{k_i + k_j + k_l}$, whereas the probability that
state \( l \) will triumph will be
\[
\frac{k_l - 2\alpha}{k_i + k_j + k_l}.
\]
The value of \( \alpha \) might depend on the relationship between offense and defense;\(^{17}\) on the particular political history between the allied states, as a history of joint operations would lead to bureaucratic infrastructures that accentuate the benefits and minimized the drawbacks of alliances; and it might depend on the ease of conducting joint operations.

When \( \alpha \) is sufficiently small, the inclusion of alliances in the model does not change the results.\(^{18}\) When membership in an alliance provides a benefit, the joining player always joins on the side of the ideologically closer belligerent, but we will see cases where it does not join and cases where the threat of joining, even in an alliance, does not deter a belligerent. If the benefit of being in an alliance is quite large, the model will become isomorphic to dyadic bargaining models, but for a significant range of values of \( \alpha \), the difference between two- and three-player bargaining games will endure. For this range of values of \( \alpha \), alliances will guarantee neither peace nor successful deterrence. For many values of \( \alpha \), the benefits of entering, even with an ally, will not outweigh the cost of conflict, and we will see dyadic war. Similarly, in some cases, the additional value of alliances will make a player willing to join a conflict but will not lead to successful deterrence, and we will see three-player war. Thus, we can derive an additional empirical implication from this model – factors that increase the returns to alliances should increase the odds of bargaining success and decrease the risk of war, but in cases where war does occur, it should have more participants: for example, we would be less likely to see war in a dispute involving two states with close military ties.

If most of the benefits of an alliance are simply the additional material capabilities that can be brought to bear against a common foe, then \( \alpha \) should be small, and this extension will have little impact on the validity of the model. If alliances are considered significantly more than the sum of their parts, then allowing formal alliances will significantly reduce the risk of conflict but this will only reduce the risk to zero when \( \alpha \) is very high. Given this outcome,

\(^{17}\) wherein technology that makes defense easier allows the lone state to defend against one enemy more easily while attacking the other

\(^{18}\) For a more complete description, and proofs of this, see the technical appendix
I would argue that extending the model to allow for formal alliances does not undermine the central findings of the model: three-player bargaining is more dangerous and war prone than two-player bargaining, and preferences are important drivers of war and peace.

In the next section, I discuss the equilibria more substantively and then examine comparative statics through simulations helping to explain when the outcome will be violent or peaceful.

4. Equilibrium Analysis

If this model involved two players bargaining over a disputed issue, there would be a unique equilibrium: peace. The inclusion of a third player substantially increases the variety and violence of possible outcomes. In the game with three players, there are four types of equilibria: peace due to agreement on the policy or successful deterrence and wars due to sufficiently divergent preferences where deterrence is absent or fails. In this section, I examine each equilibrium once I introduce a third player. Then, I discuss the comparative statics and takeaways from this model.

4.1. Equilibrium 1: Peace through Similar Interests. A key difference between two-player bargaining and bargaining with additional players is the proliferation of outside options. In the two-player game, there is only one target to attack if a proposal is rejected; if one adds an additional player is added to the bargaining process, an additional potential war is possible. Furthermore, there are two other players who each have two such options. Thus, instead of two inequalities to satisfy, there are have six. Whereas the policy range that players 1 and 2 found acceptable in the two-player game shared the same center, the expected policy outcome of a war between players 1 and 2, there may be no equivalent point of overlap in the range of policies that player 1 prefers to attacking player 2 and the values that player 3 prefers to attacking player 1.
Yet, just because a peaceful overlap of preferences is not guaranteed in a three-player bargaining game does not mean that it cannot exist. The existence of this type of equilibrium was shown in Proposition 1.2.

We see an example of this type of equilibrium, in figure 2. Here, a range of policies preferable to any possible war exists. The key difference between this equilibrium and the equivalent one in the two-player game is that this zone of agreement is not assured, and when it exists, it is significantly smaller and more constrained. When the cost of war is sufficiently high or when player preferences are sufficiently similar, we will see peace because each player finds a given compromise less painful than fighting for a better one. We might see an example of this equilibrium when we look at policy disputes among allies: Germany and France might have a markedly different ideal policy concerning spying by the NSA on foreign governments than does the United States, but the importance of this issue is subsumed by their desire to keep reasonably good relations with their ally. Thus, the cost of war would dwarf any potential benefits of achieving better espionage policies.

4.2. Equilibrium 2: War Due to Disparate Preferences, Lack of Deterrence. Our second equilibrium is a violent one. The intuition here is that the distribution of preferences makes it impossible to satisfy everyone. We could view this as the bargaining space becoming endogenously indivisible: any bargain that would satisfy two of the three players is worse than war for the third player and is thus unrealizable. In this equilibrium, the excluded player also prefers not to join the conflict. The range of policies each player prefers to dyadic conflict is disjoint, and once a dyadic conflict begins, the conflict does not grow larger. The existence of this type of equilibrium is stated in Proposition 2.

An example of this type of equilibrium is displayed in figure 4(a). In this figure, I display only the two ranges that are disjoint and driving the conflict. Even if all four other policy ranges are perfectly peace prone and well behaved, the lack of overlap between two policy ranges guarantees that the equilibria are driven by violence. In the two-player game, peace was incentive compatible, but when we have three players with divergent preferences,
finding a bargain that is mutually preferable to war is sometimes impossible for states. As an example of this type of equilibrium, consider the situation facing the Prime Minister of Pakistan trying to address both the United States and the Pakistani Taliban. If Pakistan cooperates with the United States in the war on terror, it will face serious threats of additional IEDs and suicide bombings, whereas if Pakistan tries to avoid conflict with the Taliban by ignoring the United States’ demands for cooperation, the risk of drone strikes will increase substantially.\(^19\) Whereas if they try to avoid conflict with the Taliban by ignoring the United States’s demands for cooperation, the risk of drone strikes will increase substantially.\(^20\)

4.3. **Equilibrium 3: Successful Deterrence.** The introduction of a third player in bargaining has two countervailing effects. Until now, I have focused the discussion on the first effect: third parties create regions in the bargaining space where peaceful agreement is impossible, thus potentially eliminating the prospects for peace. However, third parties can also have deterrent effects. When a player in this game chooses war, the excluded party has an opportunity to join the fray. The addition of this player can significantly reduce the benefits of war for the attacking player such that they would rather have accepted the proffered bargain. Where war and violence were (in the two-player game) strictly dominated by peaceful negotiation, in the three-player game, the threat of violence may be the only way to preserve peace. The existence of this equilibrium is stated in Proposition 3.2.

We see this potential equilibrium in figure 3(b). The original policy ranges are the same as in in subsection 4.2; there is no overlap between the bargains that player 1 prefers to attacking player 2 and those that player 3 prefers to attacking player 1. Yet, in one of those cases, the bargaining range is extended by the addition of player to a war between player 1 and 2 down the game tree, which deters player.

\(^{19}\)As the New York Times noted, “The group [the Pakistani Taliban] accused the Pakistani government of siding with the United States in its war against terror, and vowed revenge...” (Times, 2013).

\(^{20}\)The Daily Telegraph quoted a Pakistani security official saying “The talks in Chicago [on Pakistan allowing NATO convoys to use Pakistani roads] did not go well and now we see a spike in drone activity: Do the math, as our American friends might say” (Crilly, 2012).
We see this dynamic at play in a number of cases, where two of the states are great powers and the third is one of their clients. For example, in the Taiwan strait, there is no overlap between the policies that China would prefer to attacking Taiwan, assuming that the United States would not intervene, and the policies that are acceptable to Taiwan and the United States. Yet, China does not believe that it would be able to invade Taiwan without US intervention, so moderate policies become tolerable and peace in the strait endures.

4.4. Equilibrium 4: Deterrence Fails, General War. The final equilibrium of the game shares many similarities with the previous two equilibria. As in the second type of equilibrium, there are sufficiently diverse policy preferences to make harmony impossible, and as in the third type of equilibrium, we see that the willingness of excluded players to join the conflict expands the range of acceptable bargains. This deterrent effect is insufficient, and the larger bargaining ranges still fail to overlap, which may be due to the weakness of the potential intervener or the initial belligerents’ resolve with or divergence in their preferences. Given that deterrence is attempted but unsuccessful, we may see general conflict in which all three players are involved (depending on the initial offer). This equilibrium’s existence is detailed in Proposition 3.2. An example is illustrated in figure 4. Note that the threatened intervention of a third party expands the range of acceptable bargains, but there is still no possible bargain that all sides will prefer to war.

An example of this equilibrium is the invasion of Lebanon by Israel in 1982. Israel strove to replace Lebanon’s government with a friendly one, evict the PLO and isolate Syria, while Syria wanted Lebanon to remain a pliable client state (Maoz, 2006, 1988). There was no compromise which could satisfy both Israel and Syria, and Syria could neither successfully deter Israel, or allow Israel to invade Lebanon without a response. As Patrick Seale (1988) described the conflict, in his biography of the Syrian leader, “Asad and Begin, champions of irreconcilable visions, came to blows, as they were bound some time to do, over Lebanon in what was to be the goriest engagement in the struggle for the Middle East” (Seale, 1988)[p. 366].
4.5. **Comparative Statics.** While it is useful to understand that war occurs in some cases in equilibrium, the factors that drive war and peace are also relevant. I simulated results of the model for different values of each of the parameters and show how certain summary statistics drive higher or lower likelihoods of war.\(^{21}\) In particular, I examine each combination of \(c_1, c_2,\) and \(c_3\) between 0.05 and 0.4 by steps of 0.05; \(k_1, k_2,\) and \(k_3\) in integers between 1 and 4; each instance of \(s_2\) between 0 and 2 by steps of 0.05; and each instance of \(s_3\) between \(s_2\) and 2 by the same steps. There are 6,888,000 such cases, of which and 75,051 have violent equilibria. I focus on the standard deviation of preferences to show how spread out preferences are and the standard deviation of capabilities, to determine the extent to which we have either a balance or a preponderance of power.

There is a consistent positive link between variance in preferences and a higher incidence of war. In figure 5(a), the variance in \(s_1, s_2\) and \(s_3\) is plotted against the proportion of cases for which the equilibrium is war. At the same time, there are negative second order effects in the middle of the range of values, where increasing the spread of preferences somewhat decreases the risk of war. This might occur when an increase in the spread of preferences nudges a borderline deterrer into action. From these comparative statics, we can also observe that while larger spreads in preference are associated with a higher risk of war, violent conflict still occurs when the variance in ideal points is small, demonstrating that these results are not driven solely by the fact that these ideal points are unbounded. While there are some areas where the effect is non-monotonic, on the whole disputes with more closely clustered preferences are much less likely to end in violence.

As shown in figure 5(b), the relationship between capabilities and the incidence of war is consistently negative. The relationship also gains some credence as it comports with the findings

\(^{21}\)I use simulation here because of the difficulty of analytically deriving relationships. There are effectively eight variables in the model driving a binary outcome, and these variables are both interconnected and non-monotonic in their effects. For example, an increase in the cost of war for a state might reduce the probability of war by making that state more likely to compromise, but it might also increase the risk of war by removing that state’s ability to pose a deterrent threat. Similarly, the variables’ effects depend on each other: increasing the strength of a state with moderate preferences will have radically different consequences than increasing the strength of a state with extreme views on the issue.
of a number of other theoretical and empirical studies (Filson and Werner, 2002; Smith and Stam, 2004; Reed, 2003).

4.6. **Model Implications.** A major finding of the bargaining model of war has been the general irrationality of war, and this finding is diminished by three-player models. When two players are bargaining over a disputed good or issue, we can generate a bargaining range around the expected outcome of war and peacefully resolve the dispute. Incomplete information might hinder our ability to identify that range, or commitment problems might make agreeing to such a bargain difficult or impossible. However, excluding those exceptions, the bargaining model finds that war is irrational. This happy and peaceful state of affairs is partly an artifact of focusing on only two disputants. The expected outcome of a war between any two players will necessarily be better than war for each of them, but this is not so for a third party. In three-player bargaining, finding bargains that satisfy all three players is more difficult and, in certain situations, impossible (despite complete information and no commitment problems). Thus, war is a rational recourse.

Now, this finding may have interesting implications for the role of incomplete information in generating conflict. If peace is the only possible outcome when states are completely informed, then informational asymmetries either do nothing or cause war. Yet, in some three-player conflict situations, incomplete information might be the only hope for peace. If the parameters are distributed such that there is no bargain that all three players would prefer to war, then there might be a better outcome if a state overestimates the strength of moderate forces or sees its opponents as more extreme than they actually are. In this way, introducing a third party into bargaining model provides insights that are more in line with models of extended deterrence, where uncertainty is often used to create successful deterrence or to incorporate claims about the virtues of strategic ambiguity.

The model has also given us some insight into the effects of certain state characteristics on the likelihood of war. Where state capabilities are concerned, three-player bargaining can provide an additional way in which balances of power are more dangerous than preponderances
of power. This model highlights the importance of state preferences. Contrary to systemic theories of international relations, states are not black boxes; they have particular interests and preferences on a range of issues. These preferences can drive conflict; when a group of states has relatively similar preferences, then their differences will not be worth fighting over, but when these preferences are far apart, it may be impossible to find a compromise that is acceptable to all parties. We might interpret this finding as an alternative way to motivate findings about democratic and autocratic peace: if similar domestic institutions lead states to similar views on a wider range of issues, then those states will be more likely to resolve their disputes without resorting to war. We also see an interesting effect of alliances. If alliances are effective, if they are greater than the sum of their parts, then war is less likely, but the wars we observe will be larger wars.

5. Conclusion

Understanding bargaining with multiple participants has major implications for our understanding of international relations theory, empirical studies of conflict, and the conduct of peacekeeping and mediation. The model presented in this paper has demonstrated an important and little discussed scope condition for the bargaining model of war – these models have valuable insights under a dyadic framework, but if we relax the two-party assumption, a major finding of the models, the basic irrationality of war, vanishes.

By examining three-player bargaining, we are able to see a novel explanation for war based on neither incomplete information nor commitment failures. If there are more than two actors involved in a conflict, some configurations of preferences, capabilities, and resolve render successful bargaining impossible. If states have very divergent preferences, deriving an offer that satisfies all parties may be impossible, and war will become inevitable. These findings have pessimistic implications for policymakers seeking to avoid war. Incomplete information can be ameliorated with repeated interaction, third-party mediation, or iterated bargaining. Commitment problems can be avoided through institutional design or third-party guarantors.
When war is driven by incommensurable state preferences, avoiding war becomes impossible unless peacekeepers seek to alter the domestic preferences and policy of the disputants or are willing to risk war themselves in the name of deterrence.

References


Figure 2. Bargaining Space in a Three Player Game Where Harmony is Possible:

Each Line indicates the bargaining space of one player with another player. The player prefers war to the gray areas on the bargaining space, and prefers the black areas to war. The red area is the point of overlap between the 6 bargaining spaces. Note that this assumes no player will intervene in a conflict. Also note that these bargaining spaces are results of the following set of parameters: $k_1 = k_2 = k_3 = 1, s_2 = 0.5, s_3 = 1, c_2 = c_3 = 0.1, c_1 = 0.31$
(a) Player 1 and Player 3’s Bargaining Ranges if no Intervention Following an Attack

(b) Player 1 and Player 3’s Bargaining Ranges if there is an Intervention Following an Attack

Figure 3. How Intervention Effects Bargaining in the Three Player Game: Each Line indicates the bargaining space of one player with another player. The player prefers war to the gray areas on the bargaining space, and prefers the black areas to war. The red area is the point of overlap between the 6 bargaining spaces: note that in subfigure (a) there is no red space. The green space in subfigure (b) indicates how player 1’s bargaining range is extended if player 3 intervenes in a conflict between player 1 and player 2. Also note that these bargaining spaces are results of the following set of parameters: $k_1 = k_2 = k_3 = 1, s_2 = .5, s_3 = 1, c_1 = .125, c_2 = .5, c_3 = .1$
Figure 4. How Intervention Fails Effect Bargaining in the Three Player Game: Each Line indicates the bargaining space of one player with another player. The player prefers war to the gray areas on the bargaining space, and prefers the black areas to war. The red area is the point of overlap between the 6 bargaining spaces: note that in both subfigures there is no red space. The green space in subfigure (b) indicates how player 1’s bargaining range is extended if player 3 intervenes in a conflict between player 1 and player 2. Also note that these bargaining spaces are results of the following set of parameters: $k_1 = k_3 = 1, k_2 = 3, s_2 = .1, s_3 = 1, c_1 = .1, c_2 = .5, c_3 = .1$
In each of the above plots, the x-axis indicates the standard deviation of an important parameter, and the y-axis represents the proportion of equilib-rium which are violent where the parameter has those values. The points represent the mean value for those cases where the standard deviation is less than or equal to the point on the x-axis, and greater than that of the previous point—for example, for ideal points the first is the mean level of conflict for cases with standard deviation between 0 and 0.1, the second between 0.1 and 0.2, and so forth. Each line of fit is significant beyond the 0.001 level.

Figure 5.