Optimal sampling plan for clean development mechanism lighting projects with lamp population decay

Xianming Ye*, Xiaohua Xia, Jiangfeng Zhang

Department of Electrical, Electronic and Computer Engineering, University of Pretoria, Pretoria, 0002, South Africa

Abstract

This paper proposes a metering cost minimisation model that minimises sampling cost under the constraints of the required sampling accuracy of clean development mechanism (CDM) energy efficiency (EE) lighting project. Usually small scale (SSC) CDM EE lighting projects expect a crediting period of 10 years given that the lighting population will decay as time goes by. The SSC CDM sampling guideline restricts that the monitored key parameters for the carbon emission reduction quantification must satisfy the sampling accuracy of 90% confidence and 10% precision, known as the 90/10 criterion. For the existing registered CDM lighting projects, sample sizes are either decided by professional judgments or by rule-of-thumb without considering any optimisation. Samples are randomly selected and their energy consumptions are monitored continuously by power meters. In this study, the sampling size determination problem is formulated into a metering cost minimisation model by incorporating a linear lighting decay model as given by the CDM guideline AMS-II.J. The 90/10 criterion is formulated as constraints to the metering cost objective function. Optimal solutions to the problem minimise the metering cost whilst satisfying the 90/10 criterion for each reporting period. The proposed metering cost minimisation model is applicable to other CDM lighting projects with different population decay characteristics as well.

Keywords: CDM, sample size determination, energy efficiency, lamp failure rate

Nomenclature

Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\chi}(K)$</td>
<td>the cumulative sample mean up to the $K$th crediting year</td>
</tr>
<tr>
<td>$\bar{X}(i)$</td>
<td>the random variable denotes sample mean of the daily lamp energy consumption in the $i$th year</td>
</tr>
<tr>
<td>$\bar{x}(i)$</td>
<td>the value of the sample mean in the $i$th year</td>
</tr>
<tr>
<td>$\delta$</td>
<td>the $\delta$th year, $1 \leq \delta \leq I$</td>
</tr>
<tr>
<td>$\Gamma(K)$</td>
<td>the cumulative standard deviation up to the $K$th crediting year</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>the design variable</td>
</tr>
<tr>
<td>$\lambda^*$</td>
<td>the optimal solution</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>the search starting point to solve the optimisation model</td>
</tr>
<tr>
<td>$\mu(i)$</td>
<td>the true mean value in the $i$th year</td>
</tr>
<tr>
<td>$\sigma(i)$</td>
<td>the true standard deviation in the $i$th year, $\sigma(i) = \bar{x}(i) CV(i)$</td>
</tr>
<tr>
<td>$\theta(K)$</td>
<td>the cumulative true mean up to the $K$th crediting year</td>
</tr>
<tr>
<td>$a$</td>
<td>the individual meter device cost</td>
</tr>
<tr>
<td>$b$</td>
<td>the installation cost per meter</td>
</tr>
<tr>
<td>$B(i)$</td>
<td>the backup meters in the $i$th year, $B(0)=0$</td>
</tr>
<tr>
<td>$c$</td>
<td>the monthly maintenance cost per meter</td>
</tr>
<tr>
<td>$CV(i)$</td>
<td>the estimated CV value in the $i$th year</td>
</tr>
<tr>
<td>$E_B$</td>
<td>the daily energy consumption baseline (in kW h)</td>
</tr>
<tr>
<td>$E_j$</td>
<td>the daily energy consumption per lamp in the $j$th group (in kW h)</td>
</tr>
<tr>
<td>$H$</td>
<td>the annually average operating hours of the lamps</td>
</tr>
<tr>
<td>$I$</td>
<td>the number of years of the CDM projects’ crediting period</td>
</tr>
<tr>
<td>$i$</td>
<td>the counter of years, $i=0$ denotes the baseline period</td>
</tr>
<tr>
<td>$J$</td>
<td>the number of the subgroups of a project</td>
</tr>
<tr>
<td>$j$</td>
<td>the counter of the subgroups of a project</td>
</tr>
<tr>
<td>$K$</td>
<td>the counter of years</td>
</tr>
<tr>
<td>$L$</td>
<td>the rated lifespan of a kind of lamp</td>
</tr>
<tr>
<td>$lb$</td>
<td>the lower bound of the design variable</td>
</tr>
<tr>
<td>$N$</td>
<td>the lighting population</td>
</tr>
<tr>
<td>$n$</td>
<td>the sample size with population corrections</td>
</tr>
<tr>
<td>$N(i)$</td>
<td>the lighting population in the $i$th year, $N(0)$ is the baseline lighting population</td>
</tr>
<tr>
<td>$N(0)$</td>
<td>the initial sample size without population corrections</td>
</tr>
<tr>
<td>$N_j$</td>
<td>the number of devices in the $j$th group</td>
</tr>
<tr>
<td>$O_j$</td>
<td>the average daily operating hours of devices in the $j$th group</td>
</tr>
<tr>
<td>$p$</td>
<td>the relative precision</td>
</tr>
<tr>
<td>$p(i)$</td>
<td>the relative precision level in the $i$th year</td>
</tr>
</tbody>
</table>

Preprint submitted to Applied Energy

August 27, 2013
1. Introduction

CDM is a market-based mechanism under the Kyoto Protocol whereby projects in developing countries can earn tradable credits equivalent to the amount of CO₂ that are reduced or avoided. The CDM stimulates sustainable development and greenhouse gas emission reductions. In response to the climate change and global warming, a large number of energy efficiency lighting projects have been registered under UNFCCC since lighting consumes a significant amount of world energy resources, particularly, lighting consumes more than 2,000 TW h of electricity globally, which corresponds to about 1,800 million metric tons of GHG emissions per year [1]. In addition, lighting also exhibits a great potential for energy savings and GHG emission reductions. According to [2], the global cost of lighting energy is approximately $230 billion per year, of which $100 to $135 billion can be saved with today’s technologies.

The lighting energy consumption is determined by the production of two independent variables of the lamps, power and operating time [3]. Therefore, the lighting energy savings are generally achieved by either reducing the input wattage or cutting the operating time of the lamps ([4], [5] and [6]). In order to quantify the CERs for the CDM EE lighting projects, the energy savings of the lamps usually need to be impartially and transparently verified by the scientific process of M&V ([7] and [8]). The CDM general guidelines [9] and AMS-II.C [10] indicate that CER credits are calculated by the corresponding energy consumption reduction multiplied the emission factors. Normally the CDM EE lighting projects contain huge lighting population whose power varies in a wide range and operating time changes frequently. Detailed sub-metering of the lighting population is not practically feasible due to prohibitive metering cost. Therefore, sampling strategies are introduced to quantify the CER volumes with the expected accuracy cost-effectively. Specifically, the key parameters to determine the baseline and project energy consumption need to be quantified by monitoring and sampling methodologies ([11] and [12]). These sampling methodologies restrict the sampled parameters to satisfy 90% confidence and 10% precision, the so-called 90/10 criterion for most of registered CDM projects. For the 90/10 criterion, precision is an assessment of the error margin of the final estimate and confidence is the likelihood that the sampling results in an estimate within a certain range of the true values.

To guarantee the 90/10 criterion for the CERs cost-effectively, an obvious observation is to use the minimal sample sizes for the sampling plan. Theoretically, the sample sizes are determined either by frequentist methods or the Bayesian methods [13]. For instance, the frequentist approaches are applied in the studies [14] and [15] to determine the sample size while [16] and [17] adopt the Bayesian methods in choosing the proper sample sizes. Both methods use the prior information such as the required confidence and precision levels, the population of the sampling targets, the variance of the population. The frequentist methods are also referred in the CDM sampling guidelines ([11] and [12]) for the sample size determination. However, according to the PDDs of the registered CDM projects², the sample sizes for these projects are either decided

---

²Available at: http://cdm.unfccc.int/Projects/projsearch.html
by the CDM guidelines ([11] and [12]) or rules-of-thumb. The sample sizes for most of the existing CDM projects do not seem to have been determined optimally thereby unnecessary sampling expenditures are incurred. Previous studies [18], [19] and [20] have done some optimisation to minimise the sampling cost for the lighting projects. The studies [18] and [19] have proposed the metering cost minimisation models that minimise the metering cost for CDM lighting projects by optimally assigning specific confidence and precision levels to different lighting groups with different sampling uncertainties. These models are applicable and useful in optimising the sampling plan but without considering the lighting population dynamics during the CDM projects’ life cycle. The lamp population will decay due to the lamp breakage, theft or other reasons over the CDM projects’ 10-year crediting period. The sampling theory [21] indicates that the sample size can be reduced when the targeted population becomes smaller. The study [20] has considered the influence of the lighting population variation to the sampling plan and a simulation to minimise the sampling cost over a 2-year period has been provided. However, no lamp population variation model for a longer period has been incorporated in the study.

The main contribution of this study is to minimise the sampling cost for the CDM lighting projects longitudinally by the optimal determinations of the sample sizes as the lamp population varies over the CDM projects’ 10-year crediting period. For this purpose, a metering cost minimisation model is developed with the consideration of the CDM sampling accuracy requirements, the lighting population and its future variations as project proceeds, and the energy consumption uncertainties of the lamp population. In the model, a cost function that covers the meter purchasing, installation and maintenance costs of the metering system over the crediting period is formulated as the objective function. The required accuracy of each project monitoring report, which is given in terms of cumulative confidence and cumulative precision during each reporting period, is formulated as the constraints for the proposed model. Without loss of generality, the 90/10 criterion is applied as the constraint for this model. A lamp population decay model proposed by the CDM guideline AMS-II.J [22] is adopted and incorporated in both the objective function and the constraints. By solving the proposed metering cost minimisation model, the required annual sample sizes are optimised without violating the 90/10 criterion constraints whilst the sampling cost for the overall project is minimised. The advantages of the proposed model are validated by a case study of a practical CDM lighting retrofit project. In addition, this minimisation model can also be applied to other similar lighting project with different lighting population variation characteristics.

The paper is organised as follows: preliminary studies on the CDM guidelines and baseline methodologies, lamp population decay, uncertainty analysis and sample size determination methods are reviewed in Section 2. Subsequently, some essential assumptions are made in order to build the metering cost minimisation model in Section 3. Afterwards, detailed descriptions of a CDM lighting project is given as the case study in Section 4 while the optimal solutions for the case study is provided in Section 5 with a discussion of the model application. The conclusion comes at the end.

2. Preliminaries

2.1. CDM lighting guidelines and baseline methodologies

There are several approved CDM lighting project guidelines and baseline methodologies summarised in [23] such as AM0046 [24], AMS-II.C [10], AMS-II.J [22], AMS-II.L [25] and AMS-II.N [26]. The AMS-II.C offers indicative simplified baseline and monitoring methodologies for the demand-side energy efficiency activities for specific technologies such as installing new energy efficiency lamps, ballasts, refrigerators, motors and fans. The AM0046 focuses on large scale CDM lighting projects and the monitoring requirements of this methodology are very cumbersome according to [27]. The AMS-II.J is actually a deemed savings methodology that has relaxed the heavy monitoring requirements of AM0046. But the AMS-II.J generates significantly less CERs than the AMS-II.C due to a very conservative assumption on average daily utilisation of CFLs. The AMS-II.L offers guidance to the activities that lead to the adoption of EE lamps to replace inefficient lamps in outdoor or street lights. And the AMS-II.N is a guideline to the demand side CDM EE projects for the installation of EE lamps and/or controls in buildings.

For CDM lighting projects with different characteristics, different guidelines may be adopted for the CER quantification. However, the lighting baseline energy calculation approaches are found to be quite similar in all the aforementioned lighting guidelines ([10], [22], [24], [25] and [26]) as given in eq. (1)

\[
E_B = \sum_{j=1}^{J} (N_j \cdot P_j \cdot O_j),
\]

where \(N_j\), \(P_j\) and \(O_j\) are the number, power and the average daily operating hours of devices in the \(j\)th group. \(J\) is the total number of groups for a certain CDM project. \(P_j\) and \(O_j\) may be determined separately or in combination, i.e., as energy consumption. Thus, eq. (1) could be simplified into

\[
E_B = \sum_{j=1}^{J} (N_j \cdot E_j),
\]

where \(E_j\) is the daily energy consumption per lamp in the \(j\)th group. When the energy consumption baseline \(E_B\) multiplied by the number of days during the reporting period and the relevant emission factor, the baseline emission of the lighting population can be obtained. Energy consumption at the post implementation stage can also be determined by eq. (2) when apply the energy consumption of the newly installed EE lamps.

2.2. Lamp population decay modelling

A linear lamp population decay model is proposed in the AMS-II.J [22] as given in eq. (3)

\[
f(i) = \begin{cases} 
  i \times H \times \frac{100}{100 - L}, & \text{if } i \times H < L, \\
  100\%, & \text{if } i \times H \geq L,
\end{cases}
\]
where \( i \) is the counter of years; \( H \) is annual operating hours of the lamps; \( Y \) is the percentage of lamps that are operating at the rated lifetime (recommended value is 50), \( L \) denotes the rated lifespan of a kind of lamp. In the model eq. (3), when \( i \times H \geq L, f(i) = 100\% \), all lamps are deemed to be failed and no more CER will be issued for the lighting project thereafter.

2.3. Uncertainty analysis and sample size determination

According to the ASHRAE guideline [28] and IPMVP 2012 [7], the energy savings verification uncertainties can be classified into 3 categories, namely the measurement uncertainty, the modelling uncertainty and the sampling uncertainty. The measurement uncertainties usually come from the inappropriate calibration of the measurement equipment, inexact measurement, or improper meter selection, installation or operation. The modelling uncertainties are due to the improper mathematical function form, inclusion of the irrelevant variables or exclusion of relevant variables. The sampling uncertainties are resulted from inappropriate sampling approaches or insufficient sample sizes.

In this study, only the sampling uncertainties are considered since the measurement uncertainties can be reduced by using high accuracy measurement devices while the modelling uncertainties are avoidable by choosing the proper mathematic function forms and relevant variables. As provided in the statistic text book [21], the initial sample size \( n_0 \) to achieve a certain confidence and precision level of the sampling target is calculated by

\[
n_0 = \frac{z^2 CV^2}{p^2},
\]

where \( z \) denotes the abscissas of the normal distribution curve that cut off an area at the tails to give desired confidence level, also known as the \( z \)-score and \( p \) is the relative precision. For the 90/10 criterion, \( z=1.645 \) for 90\% confidence and \( p=10\% \) as the allowed margin of error in eq. (4). \( CV \) is defined as the standard deviation of the sampling records divided by the mean. \( CV \) values are between 0 and 1. If \( CV \) value is close to 0, then it indicates that the uncertainty of measurement is small. However, if \( CV \) is close to 1, then it indicates the monitored parameter has large uncertainty. \( CV \) can be estimated from spot measurements or derived from previous metering experience. If \( CV \) is unknown, 0.5 is historically recommended by [29] as the initial \( CV \). Usually more samples are required to achieve a higher confidence level and a better precision level for a given \( CV \) value. The initial sample size \( n_0 \) can be adjusted by eq. (5) [21] when the population \( N \) is a finite number. As can be observed in eq. (5)

\[
n = \frac{n_0 N}{n_0 + N},
\]

when \( N \) reduces from \( +\infty \) to 0, the sample size will become smaller.

3. Assumptions and modelling

3.1. Modelling assumptions

In this study, the following assumptions apply for the metering cost minimisation model.

(1) The lighting samples can be measured independently.

(2) The lamp population do not decay during the baseline period and the time for the project implementation can be ignored.

(3) During the reporting period, maintenance will be performed to the meters in use, but not to the backup meters.

(4) The uncertainties of the lamp population decay model are not considered.

(5) Recalling the well-known central limit theorem [30], \( X(i) \) is assumed to be subject to normal distributions, specifically, \( X(i) \sim N(\mu(i), \sigma(i)^2) \). If \( n(i) \) samples are drawn in the \( i \)-th year, the sample mean also follows a normal distribution \( \bar{X}(i) \sim N(\mu(i), \sigma(i)^2/n(i)) \) [31].

(6) \( \bar{X}(i) \)'s are independent since the samples are randomly distributed in different geographic locations.

3.2. The metering cost minimisation model

In this section, the metering cost minimisation model is built to assist the sampling plan for CDM lighting projects. This model optimally determines the annual sample sizes over the crediting period by considering the required confidence and precision levels and the lighting population decay. It is expected that the model could be applicable to CDM lighting projects with different characteristics such as different population sizes, different energy consumption uncertainties, different accuracy criterion, different crediting periods, and different reporting intervals.

To begin with, the optimisation idea is illustrated by the following example. Given a CDM lighting project with its population decays over the crediting periods and let the 90/10 criterion applies to each reporting period. For a certain 2-year reporting period, it is possible to assign 50 samples in the 1st year but only 30 samples in the 2nd year to satisfy the 90/10 criterion. Less samples are required in the 2nd year due to the lighting population decay. In this case, 50 meters must be purchased in the 1st year when the 20 surplus samples are unnecessary in the 2nd year. Alternatively, let 40 samples be monitored in the 1st year but a poor accuracy such as 70/20 is achieved. In the 2nd year, these 40 samples may result in a high accuracy such as 95/5 when the lighting population is smaller than in the 1st year. The combined accuracy over the 2-year reporting period may still meet the 90/10 criterion. When comparing the two possible solutions, the latter one requires only 40 samples to initialise the metering system instead of 50 meters, which may result in a reduction of the sampling cost for this project.

In order to maximise the sampling cost reduction in the abovementioned example, the annual sample size must be optimally determined without violating the required 90/10 criterion. Therefore, the problem is mathematically formulated as to minimise the metering cost objective function whilst satisfying the 90/10 criterion constraints. The design variables are the confidence and precision levels in the \( i \)-th year. Once the design variables are obtained, the optimal sample sizes \( n(i) \) can be determined by eq. (4) and eq. (5) with the estimated \( CV \) values.
Detailed annual metering costs over the crediting period are listed in Table 1 and the metering cost function is summarised in eq. (12). The metering cost for the baseline period includes the purchasing, installation and 3 months’ maintenance cost of \( n(0) \) meters. During the crediting period, only the maintenance cost is required for the meters in use. As the lamp population decays, the number of required meters may also decrease. Thus, if more than required meters are available, then the additional meters remain onsite for backup use. The backup meters are denoted by \( B(i) \) and

\[
B(i) = \max(B(i - 1), 0) + n(i - 1) - n(i).
\]

On the other hand, if more meters are required in the \((i + 1)\)th year than the available meters in the \(i\)th year, then some extra meters will be purchased and installed. In Table 1, \( S(i) \) is defined as follows,

\[
S(i) = \frac{1}{2} \text{sgn}(B(i)) - \frac{1}{2} = \begin{cases} 
0, & \text{if } B(i) > 0, \\
-\frac{1}{2}, & \text{if } B(i) = 0, \\
-1, & \text{if } B(i) < 0,
\end{cases}
\]

where the sign function

\[
\text{sgn}(t) = \begin{cases} 
1, & \text{if } t > 0, \\
0, & \text{if } t = 0, \\
-1, & \text{if } t < 0.
\end{cases}
\]

Let \( z(i) \) and \( p(i) \) represent the \( z \)-score and the relative precision, then the sample size \( n(i) \) is calculated by

\[
n(i) = \frac{z(i)^2 CV(i)^2 N(i)}{z(i)^2 CV(i)^2 + N(i)p(i)^2},
\]

in which

\[
N(i) = N(0) \times (1 - f(i)),
\]

where \( N(0) \) is the lighting population in the baseline period, which is the same as the number of CFL installations; \( f(i) \) is the lamp population decay as defined in the Subsection 2.2.

If the \( \bar{X}(i) \)'s are independent, then a series of the \( \bar{X}(i) \)'s over the crediting period will follow a normal distribution \( \chi(K) \sim N(\theta(K), \Gamma(K)) \), where

\[
\chi(K) = \frac{\sum_{i=1}^{K} N(i)\bar{X}(i)}{\sum_{i=1}^{K} N(i)}
\]

is the cumulative sample mean up to the \( K \)th crediting year;

\[
\theta(K) = \frac{\sum_{i=1}^{K} N(i)\mu(i)}{\sum_{i=1}^{K} N(i)}
\]

is the cumulative true mean up to the \( K \)th crediting year; and

\[
\Gamma(K) = \sqrt{\sum_{i=1}^{K} \frac{\sigma(i)^2}{n(i)} \cdot \frac{N(i)^2}{\sum_{i=1}^{K} N(i)^2}}.
\]

is the cumulative standard deviation up to the \( K \)th crediting year. Applying the \( Z \)-transformation formula

\[
z = \chi - \frac{\mu}{\sigma/\sqrt{n}},
\]

one has

\[
Z(K) = \frac{\chi(K) - \theta(K)}{\Gamma(K)},
\]

and

\[
P(K) = \frac{\chi(K) - \theta(K)}{\bar{\chi}(K)},
\]

where \( Z(K) \) is the cumulative \( z \)-score up to the \( K \)th crediting year that corresponding to a certain level of confidence. For instance, \( Z(2) \) corresponds to the combined confidence levels for the first 2 years of the crediting period. \( P(K) \) is the cumulative relative precision up to the \( K \)th crediting year. Particularly, \( P(2) \) denotes the combined precision levels for the first 2 years of the crediting period.

In summary, the metering cost minimisation model is to find \( \lambda = (z(1), p(1), \ldots, z(I), p(I)) \) that minimises

\[
f(\lambda) = (a + b + 3c) \times n(0) + \sum_{i=1}^{I} (12c \times n(i) + B(i)S(i)(a + b)),
\]

subject to the constraints

\[
\begin{align*}
Z(\delta) & \geq 1.645, \\
P(\delta) & \leq 10\%,
\end{align*}
\]

where \( I \) is the total years of the crediting period; \( \delta \) is employed to denote the \( \delta \)th year when a monitoring report to be compiled, \( 1 \leq \delta \leq I \). For instance, if it is planned to report the performance of a CDM lighting project every the second year, then \( \delta = 2, 4, 6, 8 \) and 10. Obviously, one can also let \( \delta = 1, 4, 7 \) and 10 since the reporting intervals do not seem to be restricted in any of existing CDM guidelines.

4. Case study: model application on a CDM lighting project

4.1. Backgrounds of a CDM lighting project

As given in one of the CDM PDDs [32], the project activity is to boost the energy efficiency of South Africa’s residential lighting stock by distributing CFLs free of charge to households in the provinces of Gauteng, Free State, Limpopo, Mpumalanga and Northern Cape. There are approximately 607,559 CFLs to be distributed to replace the in inefficient ICLs. The 20 W CFLs will be directly installed to replace the same number of 100 W ICLs. The CFLs with a special designed long rated life of 20,000 h provide equivalent lumen to the replaced ICLs. The walk-through energy audit results show that the daily operating consumptions of the ICLs are quite uncertain. However, the old lighting systems roughly burn 4.5 h per day on average. The removed ICLs will be stored and destroyed while counting and crushing certificates for the ICLs will be provided by a disposal company.

4.2. Monitoring and sampling plan

In both the baseline and the crediting period, the daily energy consumptions of the lighting population will be monitored and sampled. Since there is only one kind of lamps involved in either the baseline or the crediting period, it is assumed that both
the baseline and crediting period lighting systems are homogeneous and simple random sampling approach can be adopted for the sampling [12].

The proposed metering cost minimisation model will be applied to design an optimal sampling plan for this project. The model determines the optimal sample size and these samples will be randomly distributed where the baseline lamps are in use. A detailed monitoring and sampling plan is designed as follows.

(1) The expected crediting period of this project is 10 years. The monitoring reports will be compiled every 2 years post implementation of this project. The sampled parameters must satisfy the 90/10 criterion in each monitoring report.

(2) The meters will be purchased and installed during the baseline period. The daily energy consumption of the baseline lamps will be measured for 3 calendar months.

(3) The daily energy consumption of the sampled CFLs will be continuously measured during the crediting period.

(4) Meters will be installed to monitor the sampled lamp appliance individually. Once the metering devices are installed, the locations of the meters will not change. Necessary calibration and maintenance of the metering systems will be performed regularly on monthly basis.

Since the sampling targets exhibit high uncertainties, high accuracy meters with the specifications listed in Table 2 are recommended. According to [33], the key components of the metering cost include meter purchasing cost, installation cost and maintenance cost. The cost implication is also given in Table 2 as provided by a local meter company.

### Table 2: Metering device specifications.

<table>
<thead>
<tr>
<th>Categories</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage range (AC)</td>
<td>100-380 V</td>
</tr>
<tr>
<td>Current range</td>
<td>10 mA-100 A</td>
</tr>
<tr>
<td>Accuracy</td>
<td>± 0.002 %</td>
</tr>
<tr>
<td>Time resolution</td>
<td>0.5 s</td>
</tr>
<tr>
<td>Memory capacity</td>
<td>8 MB</td>
</tr>
<tr>
<td>Purchase cost</td>
<td>R 4,032</td>
</tr>
<tr>
<td>Installation cost</td>
<td>R 420</td>
</tr>
<tr>
<td>Monthly maintenance</td>
<td>R 122</td>
</tr>
</tbody>
</table>

5. Optimal solution to the case study

5.1. Initial values for the model

Now consider solving the metering cost minimisation model given in eq. (12) for the case study. Due to the nonlinear nature of the model, there is no close form solutions that can be directly applied. In this study, only numerical solutions to this model are discussed with practical initial values that are identified from the walk through energy audit.

In the objective function of the model eq. (12), the metering equipment cost including purchasing, installation and maintenance is obtained by the metering companies. The annual optimal sample sizes are determined by $\bar{x}(i)$, $p(i)$, $N(i)$ and $\text{CV}(i)$. $\bar{x}(i)$ and $p(i)$ are the design variables. $N(i)$ is calculated by eq. (9). Since metering data are not available at the planning stage, $\text{CV}(i)=0.5$ is assumed to be applicable in the crediting period. Since the metering system monitors the same target, it is also assumed that the value of annual sample mean $\bar{x}(i)$ remains constant. Thus the annual standard deviation is also constant.

The energy audit results also indicate $L=20,000$ h, $H=1,460$ h and $Y=50$. The lamp failure rates are calculated by eq. (3) and listed in Table 3.

### Table 3: CFL failure rate.

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>LFR</td>
<td>4.56%</td>
<td>9.13%</td>
<td>13.69%</td>
<td>18.25%</td>
<td>22.81%</td>
</tr>
<tr>
<td>Year</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>LFR</td>
<td>27.38%</td>
<td>31.94%</td>
<td>36.50%</td>
<td>41.06%</td>
<td>45.63%</td>
</tr>
</tbody>
</table>

In summary, the initial values to solve model eq. (12) are provided in Table 4.

### Table 4: Initial values.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meter unit price</td>
<td>$a=4,032$</td>
</tr>
<tr>
<td>Installation per meter</td>
<td>$b=420$</td>
</tr>
<tr>
<td>Monthly maintenance</td>
<td>$c=122$</td>
</tr>
<tr>
<td>CV</td>
<td>$\text{CV}(i)=0.5$</td>
</tr>
<tr>
<td>Initial population</td>
<td>$N(0)=607,559$</td>
</tr>
<tr>
<td>Reporting years</td>
<td>$\delta=2, 4, 6, 8, 10$</td>
</tr>
</tbody>
</table>

5.2. Benchmark without optimisation

In order to demonstrate the advantages for the proposed metering cost minimisation model, the metering costs for the case study without optimisation are calculated as a benchmark for comparison purpose. Without considering the optimisation for
the given CDM lighting project, the 90/10 criterion will be di-
rectly applied to decide the sample sizes for each crediting year.

The metering costs for this CDM lighting project without op-
timisation are summarised in Table 5. The CFL decay is also
considered for the solutions without optimisation. Since CDM
applies a linear CFL decay model, the survived lamp population
also follows a linear function as shown in Figure 1. It shows that
only around half of the lamps are survived at the end of the 10th
year. This suggests the required samples size at the end of the
10th year can be reduced.

As shown in Table 5, an overall metering cost of R 1,323,144
needs to be invested. It is also found that as the 90/10

criterion which

Algorithm: \( \text{interior-point} \)

TolFun: \( 10^{-45} \)

TolCon: \( 10^{-45} \)

TolX: \( 10^{-45} \)

Hessian: ['lbfgs', 20]

\[\begin{align*}
&\text{lb}: (z(1), p(1), \ldots, z(10), p(10)) \\
&\text{ub}: (z(1), p(1), \ldots, z(10), p(10)) \\
&\text{Aeq}: (z(1), p(1), z(10), p(10)) \quad (0, 0, \ldots, 0, 0) \\
&\text{beq}: (z(1), p(1), \ldots, z(10), p(10)) \quad (+\infty, 0, \ldots, +\infty, 0) \\
&\text{tol}: (0, 1, 0, 1, 0)
\end{align*}\]

From a theoretical perspective, the sample sizes should be
integral numbers for the solution. Since this study focuses on
the practical issues of minimising the metering cost, real-valued
sample sizes are used during the optimisation. After the op-
timal solution \( x^{\ast} = (z(1), p(1), \ldots, z(10), p(10)) \) is found, the ceil
function is applied to obtain the integer sample size. Table 7
gives the optimal solutions such as \( z(i), p(i), n(i) \) and the annual
metering cost.

Comparing to Table 5, it is found in Table 7 that the cumula-

tive confidence and precision levels for each monitoring report
satisfy the 90/10 criterion. In addition, the sample size is min-
imised and the overall metering cost is reduced considerably.
Specifically, the overall metering cost without optimisation is
around 1.323 million Rand. With the optimisation model, the
overall metering cost is around 0.338 million Rand. The meter-
ing cost has been reduced 74.45% with the application of the
proposed metering cost optimisation model.

5.3. Optimal solution

The MATLAB function “fmincon” is applied to find the opti-
mal solution of eq. (12). The optimisation settings of the “fmin-
con” function are shown in Table 6, where the interior-point
algorithm is chosen as the optimisation algorithm; the three
termination tolerances on the function value, the constraint vi-
olation, and the design variables are also given. In addition,
“fmincon” calculates the Hessian by a limited-memory, large-
scale quasi-Newton approximation, where 20 past iteration s are
remembered. Besides these settings, a search starting point \( \lambda_0 \)
and the boundaries of the design variable are also assigned.

<table>
<thead>
<tr>
<th>Year</th>
<th>( z(i) )</th>
<th>( p(i) )</th>
<th>( Z(i) )</th>
<th>( P(i) )</th>
<th>( n(i) )</th>
<th>Cost (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>90% 10%</td>
<td>90.00%</td>
<td>9.97%</td>
<td>68</td>
<td>367,264</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>90% 10%</td>
<td>90.00%</td>
<td>9.97%</td>
<td>68</td>
<td>99,552</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>90% 10%</td>
<td>98.00%</td>
<td>9.97%</td>
<td>68</td>
<td>99,552</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>90% 10%</td>
<td>99.56%</td>
<td>9.97%</td>
<td>68</td>
<td>99,552</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>90% 10%</td>
<td>99.98%</td>
<td>9.97%</td>
<td>68</td>
<td>99,552</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>90% 10%</td>
<td>99.99%</td>
<td>9.97%</td>
<td>68</td>
<td>99,552</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>90% 10%</td>
<td>99.99%</td>
<td>9.97%</td>
<td>68</td>
<td>99,552</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>90% 10%</td>
<td>99.99%</td>
<td>9.97%</td>
<td>68</td>
<td>99,552</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>90% 10%</td>
<td>99.99%</td>
<td>9.97%</td>
<td>68</td>
<td>99,552</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>90% 10%</td>
<td>99.99%</td>
<td>9.97%</td>
<td>68</td>
<td>99,552</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>90% 10%</td>
<td>99.99%</td>
<td>9.97%</td>
<td>68</td>
<td>99,552</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>68</td>
<td>1,323,144</td>
</tr>
</tbody>
</table>

Table 5: Sampling plan without optimisation.

Besides the optimal results listed in Table 7, Figures 2-5 pro-
vide the annual and cumulative confidence/precision levels, an-
nual adopted meters and backup meters, annual and cumulative

Figure 1: Survived lamps over crediting period.

Figure 2: Annual and cumulative confidence levels.
Table 7: Optimal sampling plan.

<table>
<thead>
<tr>
<th>Year</th>
<th>z(i)</th>
<th>p(i)</th>
<th>Z(i)</th>
<th>P(i)</th>
<th>n(i)</th>
<th>Cost (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>60.91%</td>
<td>7.38%</td>
<td>59.84%</td>
<td>7.19%</td>
<td>34</td>
<td>163,812</td>
</tr>
<tr>
<td>1</td>
<td>60.91%</td>
<td>7.38%</td>
<td>59.84%</td>
<td>7.19%</td>
<td>34</td>
<td>49,776</td>
</tr>
<tr>
<td>2</td>
<td>86.16%</td>
<td>12.74%</td>
<td><strong>90.00%</strong></td>
<td><strong>9.98%</strong></td>
<td>34</td>
<td>49,776</td>
</tr>
<tr>
<td>3</td>
<td>53.81%</td>
<td>11.17%</td>
<td>89.40%</td>
<td>10.25%</td>
<td>11</td>
<td>16,104</td>
</tr>
<tr>
<td>4</td>
<td>42.88%</td>
<td>8.78%</td>
<td><strong>90.14%</strong></td>
<td><strong>9.91%</strong></td>
<td>11</td>
<td>16,104</td>
</tr>
<tr>
<td>5</td>
<td>35.78%</td>
<td>9.34%</td>
<td>88.53%</td>
<td>9.46%</td>
<td>7</td>
<td>10,248</td>
</tr>
<tr>
<td>6</td>
<td>39.61%</td>
<td>10.70%</td>
<td><strong>90.31%</strong></td>
<td><strong>9.85%</strong></td>
<td>6</td>
<td>8,784</td>
</tr>
<tr>
<td>7</td>
<td>28.74%</td>
<td>9.03%</td>
<td>89.98%</td>
<td>9.67%</td>
<td>5</td>
<td>7,320</td>
</tr>
<tr>
<td>8</td>
<td>33.86%</td>
<td>11.03%</td>
<td><strong>90.39%</strong></td>
<td><strong>9.78%</strong></td>
<td>4</td>
<td>5,856</td>
</tr>
<tr>
<td>9</td>
<td>25.39%</td>
<td>9.30%</td>
<td>90.49%</td>
<td>9.68%</td>
<td>4</td>
<td>5,856</td>
</tr>
<tr>
<td>10</td>
<td>28.28%</td>
<td>10.74%</td>
<td><strong>90.53%</strong></td>
<td><strong>9.69%</strong></td>
<td>3</td>
<td>4,392</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>34</td>
<td><strong>338,028</strong></td>
</tr>
</tbody>
</table>

In these figures, Year 0 denotes the baseline period and Years 1-10 denote the reporting period.

In Figure 2, the dashed line (in blue) represents the optimal annual confidence levels while the solid line (in red) represents the cumulative confidence levels. Although the optimised annual confidence levels are poorer than 90%, the cumulative confidence levels satisfy the required 90% confidence during the reporting years, particularly in the Years 2, 4, 6, 8 and 10.

In Figure 3, the annual optimal precision levels are denoted by the dashed line (in blue) and the cumulative precision levels are represented by the solid line (in red). It is observed that the cumulative precision levels in the Years 2, 4, 6, 8 and 10 are always within the boundaries of 10% error band. It confirms that all the constraints in model eq. (12) are satisfied.

In Figure 4, the optimised sample size is denoted by the dashed line (in blue) and the backup meters is represented by the solid line (in red). It is found that the sample sizes generally decay as the lamp population decays. It is also observed that for each 2-year reporting period, i.e. Years 1-2, Years 3-4, the samples do not change too much. However, the sample sizes change significantly across reporting periods, i.e., across Years 2-3, Years 4-5. It indicates that the proposed model tries to balance the samples within the reporting periods in order to minimise the metering cost. It is also observed that there are backup meters at the end of the project. These meters can be removed and sold out at a lower price or be reused in other similar CDM projects.

In Figure 5, the annual metering cost is denoted by the dashed line (in blue) and the cumulative metering cost is given by the solid line (in red). The annually metering cost decays as the sample sizes decay.

5.4. Model application and discussion

The case study proves that the proposed metering cost minimisation model is very useful in designing the optimal sampling plan for a typical CDM lighting project. However, different CDM lighting projects have different initial lamp population, different lamp population variations and different monitoring report intervals. Therefore, in order to apply the proposed model flexibly to different CDM lighting projects, necessary modifications of the initial lamp population, the lamp population variation or the monitoring report intervals must be considered. For instance, the life span and usage patterns of the
lamps in different CDM projects may be different which will result in a different lamp population variation characteristics. Over the crediting period, the survived lamp population determines the sample size. The proposed model will also be applicable if incorporating a different lamp population decay model. More CFL lamp population decay models are investigated in [34]. In other cases, the reporting intervals for the project performance may be designed to be every 3 years [35]. The model is still applicable while the constraints in model eq. (12) are updated according to the specified reporting intervals.

6. Conclusion

In this study, a metering cost minimisation model is proposed to assist the optimal sampling plan design of the CDM energy efficiency lighting project. The metering cost is minimised by optimising the annual confidence and precision levels during the crediting period under the constraint of the 90/10 criterion for each monitoring report. The proposed metering cost minimisation model can be flexibly applied to other similar CDM projects. For instance, the model can be easily applied to LED retrofitting projects by adopting LED population decay models. And the proposed model is applicable to the CDM projects with different monitoring report intervals. In addition, this model can also be applied to projects with an accuracy requirement other than the 90/10 criterion.

Acknowledgement

This work is supported by the Centre of New Energy Systems and National Hub for the Postgraduate Programme in Energy Efficiency and Demand Side Management at the University of Pretoria.

References


