Radially polarized, half-cycle, attosecond pulses from laser wakefields through coherent synchrotron-like radiation

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Attosecond bursts of coherent synchrotron-like radiation are found when driving ultrathin relativistic electron disks in a quasi-one-dimensional regime of wakefield acceleration, in which the laser waist is larger than the wake wavelength. The disks of overcritical density shrink radially due to focusing wake fields, thus providing the transverse currents for the emission of an intense, radially polarized, half-cycle pulse of about 100 attoseconds in duration. The electromagnetic pulse first focuses to a peak intensity (7 × 10^20 W/cm^2) 10 times larger than the driving pulse and then emerges as a conical beam. Basic dynamics of the radiative process are derived analytically and in agreement with particle-in-cell simulations. By making use of gas targets instead of solids to form the ultrathin disks, the new scheme allows for high repetition rates required for applications.

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I. Introduction

Great efforts have recently been devoted to obtain attosecond bursts of coherent extreme-ultraviolet (XUV) and soft x-ray radiations. They open the door to exploring and controlling processes of the microcosm such as attosecond spectroscopy of bound electrons [1]. So far, established methods for producing such pulses are based on high harmonic generation from atomic gases, where the driving laser intensity is limited below 10^16 W/cm^2 [2]. High harmonic generation from solid plasma surface [3] makes use of relativistic optics (> 10^18 W/cm^2) [4] and allows for much brighter sources. These harmonics correspond to trains of attosecond pulses in the time domain, and can be used to isolate single attosecond spikes using high-pass filters [5]. By temporally rotating either the surface geometry [6] or the driver wavefront [7], individual spikes can be obtained even without filtering. Single attosecond pulses can also be produced from relativistic electron sheets formed by laser interaction with nanometer foils [8] or by nano-bunching at solid surfaces [9]. Wu et al. considered a double-foil setup irradiated by a ultraintense few-cycle laser pulse and the generation of a relativistic electron sheet from the first production foil by direct laser acceleration. Under oblique incidence, laser beam reflection at the secondary foil provides the transverse current, which emits an intense p-polarized half-cycle attosecond pulse [10].

In the present paper, we propose a new path, producing single intense attosecond pulses from underdense plasmas rather than solids. By making use of gas plasmas as a target, the condition of ultrahigh laser contrast is greatly relaxed and a bright source allowing for high repetition rates may become possible. The new scheme is based on plasma wave acceleration driven by an intense short laser pulse. It has been known that these relativistic wake waves exhibit strong accelerating and focusing fields for electrons, and promise acceleration to high energies over a short distance [11]. In most experiments the laser beam is tightly focused to reach highest possible intensity I_0 with waist W_0 ≤ \sqrt{\mu_0 \lambda_p / \pi}, where \lambda_p is the plasma wavelength and a_0 = 8.5 × 10^{-10} \lambda_0[\mu m] \sqrt{I_0[W/cm^2]} the normalized laser amplitude at wavelength \lambda_0. Plasma electrons within the laser path then pushed sideways by strong laser radiation pressure in the transverse direction (\propto \nabla \times \mathbf{E}) creating bubble-like wakes. Some electrons circling around the bubble sheath are transversely injected at the rear vertex [12] and accelerated forming a narrow bunch [13]. This is the concept of bubble wakefield acceleration [14]. An important feature relevant here is that the accelerating bunch oscillates in the wake wave forced by the focusing fields and emits bright betatron x-rays [15].

Here, we make use of a different regime of wakefield acceleration, occurring for driving laser waist larger than the wake wavelength. We refer to it as quasi-one-dimensional (quasi-1D) regime, since wake electrons are now dominated in 1D longitudinal motion, at least for those close to laser axis. Correspondingly, density crests of the plasma wave show disk-like profile as visualized in experiments [16]. Most notably, they converge into dense electron sheets when the wake is driven to high nonlinearities or close to wave breaking. These ultrathin density crests have been used as relativistic flying mirrors for laser backscattering [17]. Attosecond XUV/x-ray pulses can be produced in this way due to time compression and frequency up-conversion by Doppler shift factors of \~ 4\gamma_p^2, where \gamma_p = (1 - v_p^2/c^2)^{-1/2} with v_p the phase velocity of the plasma wave.

In the present work, we follow another line that does not require a second pulse, but leads to self-emission of an intense attosecond pulse from the dense electron sheet itself. For this to happen, we consider breaking of the above wake-wave symmetries that leads to injection of an ultrathin electron disk (or electron sheet) into the wakefield. As we shall see, the electron sheet contracts in transverse direction right after injection.
due to focusing wake fields, while boosted in energy primarily by longitudinal wakefield acceleration. The central new observation here is that a strong radially polarized, half-cycle, attosecond (RHA) pulse is emitted by the transverse currents established during the contraction phase. This coherent attosecond burst of electromagnetic radiation differs in several aspects from the wakefield based betatron x-rays normally obtained so far [15]. It is also found that wakefield acceleration plays a crucial role in amplifying the radiated fields. Radiation reaction damps and eventually saturates the fields, resulting in a high-energy (few millijoule) attosecond pulse of relativistic intensity.

The remainder of the paper is organized as follows. Section II discusses how to inject an electron sheet into a wakefield and, in particular, a self-injection scenario based on density transition is briefly summarized. In Sec. III, the generation of RHA pulses from an injected electron sheet is described using particle-in-cell (PIC) simulations. Comparisons with the betatron radiations normally obtained from bubble acceleration are discussed. In Sec. IV, a simplified model is built for the illustration of the basic dynamics of RHA generation. In Sec. V, additional multi-dimensional effects beyond the simplified model are discussed and verified using PIC simulations. Scalings of RHA generation with different laser parameters are also studied. Finally, section VI summarizes the whole paper and discusses the experimental conditions required.

II. Electron sheet injection into quasi-1D wakefield

Different ways may, in principle, be used to induce injection of electron sheets into a wakefield [18]. A direct and promising approach was recently described using density-gradient injection in a controlled manner. The details of the scheme are documented in Ref. [19], where an up-ramp density profile followed by a plateau is employed. Wave breaking then occurs sharply at the density transition position, leading to a sudden longitudinal injection into the quasi-1D wakefield. This is in contrary to the transverse injection that normally occurs quasi-continuously in the bubble regime [12, 14]. Notice that longitudinal injection but in a gentle way has been observed in experiments [20]. The key elements of the present sharp longitudinal injection are summarized as follows. First, along the up-ramp, the first few wake wave periods trailing after the driving laser can travel at superluminal phase speeds for high enough density gradients. It is also valid for high wave nonlinearities, so that the density wave crests can be stably compressed into dense electron sheets without premature injection [19, 21]. The free of injection eventually terminates as the wake wave propagates into the following density plateau region, where the wave’s phase velocity falls below the light speed. As a result, a major part of the sheet electrons satisfying the injection threshold are trapped as whole in a very short time scale. They typically form an ultrathin (10s nanometers thick corresponding to attoseconds in duration) overcritical dense electron disk that accelerates in the wakefield.

III. Attosecond pulse generation via particle-in-cell simulation

The evolution of an injected electron sheet in a wakefield and the associated radiation flash are first illustrated in Figs. 1(a) and 1(b). It is seen that the sheet contracts in transverse direction to a smallest diameter in tens of laser periods right after the injection, while accelerating longitudinally up to highly relativistic $\gamma$ factors (compare with Fig. 4). It is in this short time interval that the attosecond pulse is emitted. As found in Fig. 1(b), the pulse is first attached to the electron sheet, but then propagates along a cone, while the electrons move closer to the axis in a channel of almost constant diameter.

These results are obtained from 2D PIC simulations using the code OSIRIS [22]. As a demonstration case, the controlled self-injection scenario [19] is employed to induce the sheet injection. The simulation parameters are detailed as follows. A linearly polarized (along $z$ direction), 18 fs (FWHM of intensity) laser pulse of peak intensity $I_0 = 7.7 \times 10^{19} \text{ W/cm}^2$ irradiates an underdense plasma slab with a density ramping-up front of 65$\mu$m up long. The plateau density is $n_0 = 0.04 n_e$, with $n_e = 1.74 \times 10^{21} \text{ cm}^{-3}$ the critical density for laser wavelength $\lambda_0$ = 0.8 $\mu$m. A laser waist of $W_0$ = 17 $\mu$m is used and large enough for the formation of wide electron sheets in the quasi-1D regime. An electron temperature of $T_e = 20 \text{ eV}$ is initialized to mimic field ionization by pre-pulses. In the simulation, spatial grids of $\Delta x \times \Delta y = 158 \times 25$ per square and time steps of $\Delta t = 0.00625 \text{ \mu m/c}$ are chosen and fine enough to resolve the attosecond features; we have checked that even higher resolutions give nearly the same results.
We now analyze the radiative process in general. The injected electrons are diverted by focusing wake fields toward the central axis according to $dβ_⊥/dt ≈ ε_⊥/γ$. Here, $β_⊥$ is the normalized transverse velocity, and time $t$ and focusing fields $ε_⊥$ are, respectively, normalized by $ω_0$ and $E_0 = meω_0/c$ with electron mass $m_e$, elementary charge $e$ and laser frequency $ω_0 = 2πc/λ_0$. Near the center axis, the focusing field, $ε_⊥ ∝ τ_0 r$, is almost linear in both radius $r$ and ambient density $n_0$. For the present conditions, the betatron strength parameter [23] of trapped electrons $K = γk_β r_β$ typically amounts to a few tens, where $k_β = 2π/λ_β$ with $λ_β = (2γ)^{1/2}λ_p$, the betatron wavelength and $r_β$ is the amplitude of the betatron orbit. Thus these electrons perform synchrotron-like motion with curvature radius $ρ ≈ λ_0 γ/(2πε_⊥)$ and emit broad-band radiation with cut-off frequency $ω_⊥ ≈ 3γ^3ε_⊥ρ^{-1}$ [24]. There are a couple of new features compared with normal betatron radiations [15]. First, the RHA pulses are produced at the initial contraction phase, corresponding to subcycle (or more accurately, a quarter of cycle) of betatron oscillation, during which the radiation generation can be modeled with relativistic electron sheets. Second, the high density of the electron sheet, typically larger than the critical density $n_s ≥ 10^{18} cm^{-3}$ [19], enables synchrotron-like radiation in a coherent manner. Coherence occurs provided that sufficiently large number of electrons reside in a volume of the radiation wavelength dimension in the rest frame of electrons. For the present case of self-emission, it requires the sheet density to satisfy

$$n_s ≥ 10^{18} cm^{-3},$$

(1)

where radiation at the cutoff wavelength $λ_σ = 2πc/ω_⊥$ is assumed. Considering that $ε_⊥ ∼ 0.1$, this criterion can be readily met even for $γ ∼ 100$. As a result, the coherency is well preserved at the initial contraction phase, during which the trapped sheet electrons are only accelerated up moderate energies (less than 50MeV in the present case). After that the sheet electrons are confined within a small radius and shall emit normal betatron x-rays which are incoherent and orders of magnitude weaker in intensity. Third, due to axial symmetry and inward acceleration of disk electrons, radially polarized half-cycle electromagnetic pulses are produced during the contraction. Here, owing to the ultrarelativistic feature of sheet electrons, the dephasing time arising from velocity mismatch between the coherent radiation and the electron sheet is much longer than the contraction period (i.e., from injection to focused on the laser axis), which is typically a few tens of laser periods as shown in Fig. 1(a). Therefore, the radiated pulse is almost overlapped with the sheet profile during the contraction phase and naturally has an attosecond duration. It is interesting to mention that radially polarized half-cycle pulses have also been recently demonstrated from thin photococonductors, where a sequence of annulus microelectrodes are fabricated on a wafer surface to induce radial currents [25]. However, due to the static wafer plane, only pulses of picosecond duration are generated, determined by the lifetime of the radiating currents.

These features are well illustrated by the sample simulation. Figure 2(a) records how the peak field $B_nz/B_0$ of the RHA pulse evolves with time, where $B_0 = m_eω_0/e$ is the normalizing field. Combining with Fig. 1(b), we see that the attosecond pulse emerges in three stages: field amplification during initial contraction, beam crossing on axis, and finally annular beam propagating along a cone. At the stage of beam crossing, the emission is focused to a maximum of $B^mz/B_0 ≥ 18$, indicated by the shaded region in Fig. 2(a). This corresponds to a peak intensity greater than $7 × 10^{20} W/cm^2$, almost 10 times larger than the incident driving pulse. After beam crossing on axis (e.g., $t > 130 µm/c$), the upper and lower branch of the pulse propagate separately in space and the actual peak field shows up as $B^mz/B_0 = 4.5$, corresponding to $4.3 × 10^{19} W/cm^2$ in intensity or $∼ 2 × 10^{13} V/m$ in field strength. It then slowly decays due to diffraction.

Figure 2(b) shows how the RHA pulse propagates at later time in the wake wave. It is found that the initial quasi-1D wake is evolving into a complex broken-wave pattern due to laser evolution [26]. The half-cycle pulse as short as 120 as

![Figure 2](image-url)
[see Fig. 2(c)] is propagating inside the density bubble, but deviated from the center axis. The corresponding spatial spectrum of the pulse given in Fig. 2(f) shows clearly two branches, each with a small divergence. The line-out along one of the branches given in Fig. 2(e) shows a broad-band XUV spectrum extending up to \( k/k_0 \approx 25 \) or about 40 eV in photon energy, where \( k_0 = 2\pi/\lambda_0 \). In the present simulation case, the annular beam carries a total energy of \( \sim 12 \) mJ, which is about \( 2 \times 10^{-10} \) of the incident laser energy.

The RHA generation is found to be insensitive to finite thermal temperatures of the ambient plasma electrons. The emission profile hardly changes when increasing \( T_e \) up to 100 eV as depicted in Fig. 2(c). Also shown in this subfigure is the resulting emission profile when switching the polarization direction of the driving pulse from \( z \) to \( y \) (within the simulation plane) axis. Few differences are observed, indicating that the polarity of the attosecond emissions is independent of the driving laser, but only determined by the transverse contraction motion of the sheet electrons. We have confirmed this by using 3D PIC simulations (not shown here) that the annular attosecond light beam is uniform in intensity azimuthally, corresponding to radial polarization. Notice that full-scale 3D simulations for the present configuration generally cause extremely heavy computational loads due to the wide box dimensions and high resolutions required for both longitudinal and transverse directions, so that they have to be conducted with much lower resolutions compared with the above 2D runs. However, they confirm very well the half-cycle profile and the radial polaron of the attosecond emissions.

IV. Simplified model for attosecond pulse generation

The basic dynamics of RHA generation can be derived from a simplified model. It assumes that a flat, monoenergetic electron sheet of delta-like density profile \( n_s = \delta(\rho/\sigma_y) \) with finite areal density \( \sigma_y = \int n_s d\rho \) and no initial transverse momentum is injected into a wakefield of uniform accelerating and focusing fields. Coherent radiation from the electron sheet is then described by the 1D wave equation \( (\partial^2/\partial x^2 - c^{-2}\partial^2/\partial t^2)E_{y,r} = e_0^{-1} c^{-2} \partial J_{y,r}/\partial t \), where \( J_{y,r} = -e\beta_{y,r} n_s \beta_{y,r} \) is the radiating current with \( \beta_{y,r} = (\sigma_y/\epsilon_0) \beta_{y,w} \). The velocity integrated over a short transverse acceleration \( 10 \) is evaluated at the saturation instant. It leads to the saturated field

\[
E_{y,r} = E_{y,w}/\tan \theta + E_{y,w},
\]

where \( \tan \theta = \beta_{y,w}/\beta_{y,r} \) is evaluated at the saturation instant. It is clear from the above analysis that wakefield acceleration has played a crucial role in RHA generation. At this point, the present case differs from the half-cycle pulse generation from the double-foil scheme, where the relativistic electron sheet emerging from the second foil is almost fixed in \( \gamma \) and is supposed to radiate immediately the strongest fields \cite{10}. Equation (4) further suggests that \( E_{y,w}^{sat} \) can be large provided that \( \tan \theta \approx \beta_{y,w} \) is small initially and, in an ideal case, the saturated level can also be maintained for a long period.

V. Additional multi-dimensional effects

There are a couple of effects not yet included in the above simplified model where a flat electron sheet is assumed. In the sample simulation, the injected sheet is not perfectly flat but slightly curved as seen in Fig. 1(a). Owing to the sharp longitudinal injection, a curved wake wavefront \cite{16, 17} will directly map into the initial spatial profile of the injected electron sheet. More importantly, the injected electrons will also
inherit certain transverse velocities. These will cause transverse contraction during propagation even without the focusing wake fields, and the radiative process is ultimately limited by deformation of the sheet profile.

To see qualitatively what the initial transverse velocity distribution is and how it scales with laser parameters, we analyze single electron dynamics in a wakefield. To the zeroth-order, the normalized transverse velocity changes as 
\[
d\beta_y/dt = (1/\gamma)(-E_{y,w} + \beta_x B_{z,w}),
\]
where \(B_{z,w}\) is the quasi-static magnetic fields generated in a laser wake [27], and \(\gamma\) is the Lorentz factor averaged over laser cycles. The laser ponderomotive force is weak in transverse direction and not included. For the case of sharp injection, the initial velocities of sheet electrons are equal to that shortly before trapping. Considering a cold start for plasma electrons, the transverse velocity at the apex of the first wake wave crest is given by 
\[
\beta_y = -\int_0^t (1/\gamma)E_{y,w} dt,
\]
where the contribution from \(B_{z,w}\) is omitted since \(\beta_x\) changes sign during a plasma oscillation. As a consequence, the initial transverse velocities are mainly determined by the transverse wake fields \(E_{y,w}\), so larger \(E_{y,w}\) generally results in larger \(\beta_y\).

The electron density of the wake wave when approaching the density plateau is plotted in Fig. 3(a) and the corresponding transverse wake fields \(E_{y,w}\) in Fig. 3(b). The transverse line-outs of \(E_{y,w}\) along \(x = 63\ \mu\text{m}\) are given in Figs. 3(c) and 3(d) for different laser parameters. It is found that they share the similar profile with a maximum at \(y = W_0/\sqrt{2}\). Notice that this sort of distribution obtained in the present highly non-linear quasi-1D regime is similar to that of a linear wave, i.e., 
\[
E_{y,w} \propto y \exp(-y^2/W_0^2),
\]
if a Gaussian (transverse profile) driving laser is used [23]. Following the above qualitative analysis and considering that the radius of injected electron sheet normally satisfies \(r_s \leq W_0/\sqrt{2}\) [19], the sheet electrons at larger radii will have larger initial transverse velocities. As we shall see, this velocity distribution is useful for the interpretation of a couple of simulation results beyond the above simplified model.

Below we adopt particle tracking to illustrate these additional effects. A group of electrons uniformly picked from the lower half of the sheet are tracked. As seen in Figs. 4(a-c), the electrons are injected at about \(t = 74\ \mu\text{m}/\text{c}\) and focused to the axis at about \(t = 102\ \mu\text{m}/\text{c}\). During this period, these electrons develop a broad \(\gamma\) spread primarily due to longitudinal injection dynamics. Figure 5 shows the trapped electron sheet during the contraction phase. It is found that the sheet could be treated as a sequence of sub-layers, each of which is monochromatic in energy and can be well described by the simplified model shown above. As we have pointed out, the overcritical density and ultrathin feature of the sheet ensure the coherency of the overall atto-second emissions. Figure 4(d) further shows snapshots of the propagation angles \(\theta \simeq (180^\circ/\pi) \arctan \beta_y\) of these electrons at different times, either shortly before injection or during the contraction phase. It is seen that the corresponding \(\beta_y\) values are larger for larger radii and they further increase slowly with time due to transverse acceleration. This velocity distribution is in agreement with the above qualitative analysis.

The transverse velocity distribution suggests that the disk-like electron sheet can be further subdivided into a sequence of ring-shaped segments defined by their \(\beta_y\) values. Applying the simplified model to each ring segment and rotating the coordinate frame by angle \(\theta \simeq \arctan \beta_y\) with respect to the normal \(x-y\) axis, the resulting pointing direction of the radiated electric fields will deviate somewhat from \(y\) axis; see the schematic drawing in Fig. 1(b). An evidence for the angle deviation is given in Fig. 2(d), where a sharp negative \(E_z\) field shows up at the on-axis overlapping joint during the stage of beam crossing. Moreover, the fields emitted from segments
This is because the sheet injection requires high enough producing the most intense field) exists as seen in Fig. 6(b). An optimal value of $a_0$ while keeping $W_0 = 17 \mu m$ and $n_0 = 0.04 n_e$ fixed.

VI. Summary and discussions

In conclusion, we have identified a novel attosecond burst of coherent synchrotron-like radiation from laser wakefield acceleration. The attosecond feature does not derive from solid surface layers [9] or ultrathin solid foils [10], but arises from intrinsic features of nonlinear plasma waves, namely steepening and breaking. It makes use of quasi-1D wake wave breaking, allowing to trap a dense electron sheet into the wakefield. The injected electron sheet contracts radially by focusing wake fields, while boosted in energy primarily by longitudinal wakefield acceleration, and is found to emit an intense half-cycle attosecond pulse. Pulse energies over 10 mJ can be obtained with the laser-to-RHA conversion efficiency exceeding $10^{-3}$. Besides being a bright source for attosecond applications, this burst will also provide useful diagnostics for wavebreaking dynamics [28] in the highly nonlinear quasi-1D regime and corresponding electron sheet injection [19].

It is worthwhile to emphasize that the coherent half-cycle attosecond pulse emerges at the initial contraction phase of the electron sheet after injection. After completely focused to the axis, these electrons are confined within a smaller radius [see Fig. 1(a) and Fig. 2(b)] due to wave evolution and can transport for a long distance with highly relativistic energies. They perform significant betatron oscillations and shall also emit incoherent keV-level x-rays as those normally found with wakefield accelerated electrons [15]. Therefore, in real experiments, the present scheme will deliver both a coherent attosecond XUV pulse laterally and an incoherent x-ray beam in the forward direction. Since the number of trapped electrons in the quasi-1D regime with controlled injection can be made higher (such as nC) [19], the present configuration may also provide the possibility for brighter betatron radiations with better control.

We further have checked that the RHA generation is robust against changing laser and plasma parameters. Following Figs. 3(c) and the analysis therein, larger focal spots of the driving pulse lead to smaller initial transverse velocities of the injected electron sheet, thus higher field amplitudes are expected according to Eq. (4). This is verified in Fig. 6(a), where $B_z^{\text{max}}$ and $\theta_0$ scale almost linearly with the focal spot $W_0$. While scanning the laser amplitude $a_0$ for fixed plasma density of $n_0 = 0.4 n_e$ and focal spot of $W_0 = 17 \mu m$, an optimal value of $a_0 \sim 6$ for RHA generation (means producing the most intense field) exists as seen in Fig. 6(b). This is because the sheet injection requires high enough driving laser amplitudes [19]; while for too strong drivers, the quasi-1D wake excitation will be violated for fixed laser focal spot. The scaling dependence with the plasma density $n_0$ becomes even more complicated, because the laser conditions have to be adjusted accordingly to make the sheet injection possible.

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[18] An electron sheet may be externally injected into a broad wave wave driven by petawatt lasers. Possible sheet production methods were proposed, for example, in Refs. [8]