Spontaneous Generation of Self-Organized Solitary Wave Structures at Earth’s Magnetopause

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Spontaneous formation of solitary wave structures has been observed in Earth’s magnetopause, and is shown to be caused by the breakup of a zonal flow by the action of drift wave turbulence. Here we show matched observations and modeling of coherent, large-scale solitary electrostatic structures, generated during the interaction of short-scale drift wave turbulence and zonal flows at the Earth’s magnetopause. The observations were made by the Cluster spacecraft and the numerical modeling was performed using the wave-kinetic approach to drift wave-zonal flow interactions. Good agreement between observations and simulations has been found, thus explaining the emergence of the observed solitary structures as well as confirming earlier theoretical predictions of their existence.

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Soliton formation is a common phenomenon in the evolution of nonlinear waves and turbulence. Well-known examples are soliton formation in nonlinear optics and solitary waves in fluids and plasmas [1,2]. In the study of magnetically confined plasmas such as tokamaks, the interaction of zonal flows and drift mode turbulence, described by the Charney-Hasegawa-Mima equations [3–6], can cause the generation of solitary wave structures [7]. Similar phenomena occur in the coupling between Rossby waves and zonal flows in planetary atmospheres [8].

Recent observations by the Cluster satellites at the magnetopause [9–11], which is the boundary separating the shocked solar wind and Earth’s magnetosphere, demonstrate the existence of electrostatic solitary wave structures moving down the ion density gradient associated with this boundary. These structures coincide with much higher frequency modes around the lower-hybrid drift frequency, and theory suggests that the two are associated with each other [7]. In this Letter we demonstrate that the low-frequency electrostatic solitary structures originate from the breakup of zonal flows. These zonal flows are driven by the higher frequency drift modes through wave collapse [12] of the drift modes. We have applied, for the first time, the numerical model of the modulational interaction of drift modes and zonal flows to real experimental data. Numerical simulations of the wave collapse instability of drift waves, adapted for the Cluster context, show good agreement with the observations. In both theory and simulations, the drift mode turbulence exhibits strong wave collapse leading to spontaneous formation of solitary wave structures which move rapidly down the magnetopause plasma density gradient, penetrating into the magnetosphere to different depths at different sampling times.

The observations analyzed here were made by the fleet of four ESA Cluster spacecraft [9], which provide unique, multipoint measurements of the Earth’s magnetosphere. Measurements were taken during an outbound pass through the magnetopause boundary layer, which contains a current layer that separates the shocked solar wind plasma, the magnetosheath, from plasma trapped by the closed magnetic field of the Earth, the magnetosphere. The event we analyze occurred at the magnetopause boundary layer on 30 March 2002, and has been reported by Keyser et al., [10] and Silin et al. [11], who identified the electric field turbulence within the current layer to consist of lower-hybrid drift waves. The orbit and configuration of the spacecraft are given in Fig. 1, which shows that, during the event, Cluster moves from the tail lobe of the magnetosphere into the magnetosheath. As indicated in the figure, the spacecraft were on an outbound pass through the magnetopause when the event described here occurred. Figure 2 displays an overview of the measurements taken by Cluster [9,11]. Clockwise, the top four panels show the component of the magnetic field parallel to the boundary (which defines the extent and maximum value of the current layer), the background plasma density (which defines the boundary between the magnetosheath and magnetosphere plasmas), and the ac (20–180 Hz) and dc (0–20 Hz) components of the boundary normal electric field. The framed region of interest in the electric field graphs is shown magnified in the two panels at the bottom of the figure, clearly showing solitary waves in the dc field, and modulated wave packets in the ac field.

The Cluster observations displayed in Fig. 2 were taken on 30 March, 2002, between 13:11:40 and 13:11:47 UT. The field data have been taken by the EFW (electric field) [13] and FGM (magnetic field) [14] instruments of the Cluster spacecraft, while the plasma density has been derived from the spacecraft potential [15]. Note that the electric field signal for C1 (black curves in Fig. 2) was unavailable when the observations were made. The zero-
crossing of the $B_L$-component of the magnetic field took place at 13:11:45 UT for spacecraft C1, and defines the instant of the spacecraft entering the magnetopause boundary layer from the magnetosphere, as shown in the top left frame of Fig. 2 (with the magnetosphere on the left, magnetopause on the right). In order to line up the boundary layer crossings of all four spacecraft and to highlight the boundary layer traversed by each spacecraft, a time delay has been imposed on signals provided by C2 (red, $-2.1 \text{ s}$), C3 (green, $-3.1 \text{ s}$) and C4 (blue, $+0.5 \text{ s}$) to match them to C1 (black). This time delay also takes care of systematic errors in the timing of the individual spacecraft, which otherwise amount roughly to 0.1 s.

In the following, the plasma electron temperature is denoted by $T_e$, the ion (proton) mass by $m_i$, the ion sound speed by $c_s = \sqrt{T_e/m_i}$, the Earth’s geomagnetic field by $B$, the ion gyro-frequency by $\Omega_{ci} = eB/m_i$, where $e$ is the magnitude of the electron charge, and the ion sound gyroradius by $\rho_s = c_s/\Omega_{ci}$. At the Earth’s magnetopause, we have $T_e \sim 1 \text{ MK}$ and $B \sim 120 \text{ nT}$, so $c_s = 90 \text{ km/s}$, $\Omega_{ci} = 12 \text{ rad/s}$, and $\rho_s = 7.5 \text{ km}$.

The observations displayed in Fig. 2 (second panel left) are discussed from right to left to better follow the penetration of the solitary structures into the magnetosphere. The figure shows that there is the excitation of electrostatic (ES) turbulence in the dc electric field with a dominant wavelength of 1.5–2 km or $0.2–0.25 \rho_s$. This ES wave, which will be shown to correspond to a zonal flow, is observed at the edge of the magnetosheath plasma, where the plasma electron density $n$ drops from about 60 (magnetosheath) to 6 cm$^{-3}$ (magnetopause) over about 75 km, or $10 \rho_s$. This corresponds to a relative density gradient $(\rho_e/n)\nabla n$ of up to 0.9. The slow ES wave mainly occurs in regions where $(1/n)\nabla n$ is not too large; it does not grow in regions where $n$ is small [and thus $(1/n)\nabla n$ is large]. In the magnification of the highlighted region (bottom panel left), a solitary wave structure breaking away from the main ES wave is indicated by arrows. From the known separation of the spacecraft and the order in which they crossed the magnetopause, we infer that this structure, with an accompanying drift wave packet, moves down the gradient into the magnetosphere at a speed of roughly 8–9 km/s, or...
boundary. The initial drift mode distribution is homoge-
ously observed by the Cluster satellites at the magnetopause
with a density profile modeled after the density profile
used a broadband distribution of drift waves in a plasma
can be found elsewhere [7]. In our simulations, we have
modes. A detailed description of the numerical model
plementation, this distribution is approximated by a collec-
tion of macro-particles representing individual wave
packets. Moving wave packets can be observed in this signal; their position and speed coincide
with those of the solitary structures in the dc field. These
wave packets maintain their coherence during the passage
of all four Cluster spacecraft, while different wave packets
are seen to move independently of each other. The ob-
served breakup of the ac turbulence into wave packets
follows from the nature of the wave-collapse instability
[12] of electrostatic drift modes. The dimensions of these
wave packets are set by the zonal flow characteristic wave
length, which is of the order of $\rho_s$, in agreement with the
observed structure size.

Numerical simulations have been carried out to investi-
gate the mechanisms governing the emergence and
propagation of the aforementioned solitary structures. We
have used the so-called wave-kinetic approach [16–19], as
it is particularly suited to describe broadband turbulence in
the spectrum of the high-frequency wave components. It
also allows one to follow the propagation of individual
spectral modes, thus providing deeper insight into the wave
evolution.

The numerical code used for our simulations has been
based on the wave-kinetic approach to drift wave-zonal
flow interactions in magnetized plasmas [6,7,20,21], as
found in, e.g., tokamaks. This approach is centered around
the wave mode density $N(t, x, k)$, of which the evolution is
given by a Boltzmann-like equation. In the numerical im-
plementation, this distribution is approximated by a collec-
tion of macro-particles representing individual wave
packets. A detailed description of the numerical model
can be found elsewhere [7]. In our simulations, we have
used a broadband distribution of drift waves in a plasma
with a density profile modeled after the density profile
observed by the Cluster satellites at the magnetopause
boundary. The initial drift mode distribution is homoge-
neous in $(x, y)$-space, and Gaussian in $(k_x, k_y)$-space ($k =
(k_x, k_y)$ denotes the drift mode wave vector), with a mean $k$
value of $3/\rho_s$ and a spread of $1/\rho_s$. Simulation results are
displayed in Fig. 3: the electrostatic field $E$, representing
the zonal flow, and (the distribution of) the longitudinal
wave numbers $k_x$ of the drift modes, representing the fast
drift wave turbulence, are plotted versus the longitudinal
coordinate $x$. The stacked plots are therefore the numerical
counterpart of the sampling of the magnetosphere-
magnetopause region at different times by the individual
Cluster spacecraft, and correspond directly to the bottom
panels of Fig. 2.

In the simulation results, we first observe the excitation
of a dc electrostatic wave, the zonal flow, at the plasma
edge through the wave-collapse instability of the drift
modes. This zonal flow has a wavelength of about $(0.5 -
0.7)\rho_s$, which is within a factor of 2 of the observed value
of about $0.25\rho_s$. Zonal flows can only grow when the
imaginary part of their frequency, denoted by $\gamma$, is nonzero
and in regions where there is good resonance between
fluctuations in the drift mode density and the zonal flow.
Using Ref. [6], the first condition translates to $3k_x^2 < 1 +
k_{\parallel}^2$, while the second (resonance) condition requires the dia-
magnetic drift speed $V_d = (-c_s\rho_s/n)\nabla n$ to satisfy $|V_d| < |
\gamma/q| = |e\Phi/(k_B T_e)|(1 + k_x^2 + k_{\parallel}^2)^{3/2}(1 - 3k_x^2 + k_{\parallel}^2)^{1/2}/
(2|k_{\parallel}|)$, where $q$ denotes the zonal flow wave number. This
explains why initially no zonal flow develops in regions
where $n$ is small, both in observations and simulations,
because $V_d$ is too large there and there is no resonant
coupling between the drift modes and the zonal flow.

![Image](348x274 to 361x286)

![Image](355x360 to 366x772)

FIG. 3 (color). Simulation results for a thin magnetopause,
where the background plasma density profile has been chosen
to match the sudden rise in the plasma density at the magneto-
pause boundary. Snapshots of the slow electrostatic field (rep-
senting the zonal flow) and the drift mode phase space through
fluctuations in the drift mode density and the zonal flow.

These plots show the slow electrostatic field (red) and the back-
ground plasma density (black), while on the right the drift mode
phase space can be seen. The excitation of a zonal flow through
the modulation instability only occurs for larger $x$, where the
relative density gradient is shallow. A solitary wave structure
breaking off the main zonal flow and drifting down the density
gradient is indicated by arrows.
This explains both why the zonal flow growth is largest for smallest inverse scale length $1/L_n = V_d/(c_s \rho_s)$, and why the zonal flow tends to propagate towards steeper density gradients. For a fixed drift wave amplitude, large density gradients suppress the instability.

The simulations show solitary structures breaking away from the main zonal flow region and propagating independently into regions where no zonal flow developed earlier, retaining their identity for quite a long time. The size of these structures is about $(0.7 - 1.0) \rho_s$, which is close to the observed size of $(0.8 - 0.9) \rho_s$. As in the observations, the structures are stretched during propagation. Their speed increases from 0.02$c_s$ to 0.05$c_s$, which is within a factor 2 from the observations, as they move further down the density gradient. In the $(x, k_x)$-space for the drift modes, there is clumping of drift modes as a result of the wave-collapse instability. Quasiparticles from the same clump stay together for a long time, while the clumps may propagate independently from each other. These effects are all directly seen in the observations as well.

We can use the above resonance condition to estimate a threshold value for the instability in the observations. From the spacecraft data, we note that there is no instability for $\rho_s(\nabla n)/n \approx 0.2 \pm 0.1$. Inserting this into the resonance condition, this requires the drift mode amplitude to satisfy $e \Phi/(k_B T_e) \approx 0.02 \pm 0.01$. The observed amplitude of $e \Phi/(k_B T_e) \approx 0.025$ ($\sim 2$ Volts) falls well within this interval.

As explained in Ref. [7], the formation of these solitary structures can be explained from the interplay between the $\mathbf{E} \times \mathbf{B}$ drift $V_0 \propto \partial \Phi/\partial x$ (where $\Phi$ is the zonal flow potential) and the diamagnetic drift $V_d \propto -(1/n_0) \partial n_0/\partial x$, where $n_0(x)$ denotes the equilibrium plasma density. The $\mathbf{E} \times \mathbf{B}$ drift follows the perturbation in $\Phi$ and causes the solitary structure to stay together, trapped in a period of the zonal flow, while the back action that the drift waves exert on the background plasma tends to enhance the zonal flow and thus improve confinement of the drift waves. On the other hand, the diamagnetic drift acts in the same direction for all drift modes with the same sign of $k_y$, and increases with the relative density gradient. This drift causes drift modes on a density slope to move apart, thus opposing the effect of the $\mathbf{E} \times \mathbf{B}$ drift. Initially, the diamagnetic drift only causes clumps of drift modes to move away from each other and towards regions with even steeper density gradients, explaining the observed behavior of the solitary structures. However, when the relative density gradient becomes sufficiently large, the structures themselves are ripped apart. All these aspects of the drift wave-zonal flow interactions are seen in both observations and simulations. This good correspondence proves that the dc electrostatic waves in the observations are indeed zonal flows, and explains the mechanisms behind the solitary structures seen in the satellite observations.

In conclusion, we have studied Cluster observations displaying the spontaneous emergence of coherent solitary structures from broadband ac turbulence at the magnetopause boundary layer. The unique capabilities of the four Cluster spacecraft have allowed the identification of the solitary structures as separate entities. The study of their evolution, as they propagate down the plasma density gradient associated with the magnetopause and penetrate the magnetosphere, has been achieved as a result of the sequential sampling by each spacecraft. Numerical simulations have provided the interpretation of these structures in terms of a nonlinear phenomenon known as wave collapse. The drift modes turbulence, also observed by Cluster, is the driving force behind this phenomenon.

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